Proceedings of the 2023 International Conference on Mathematical Physics and Computational Simulation
DOI: 10.54254/2753-8818/28/20230389

A competitive infectious transmission model

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Abstract. Compartmental disease transmission models are widely used to model state transmission in infectious diseases, using differential equations to model the change in the number of units in different states over time, and recently has produced significant practical implications in many downstream fields. However, inspired by the transmission of rumors in social media, we note that the previous compartmental transfer models neglect the "competitiveness" during the transfer process, that is, the "infection" of people with positive and negative opinions to "susceptibles" or even people with opposing views. To tackle the above issues, in this paper, we propose a novel competitive infectious transmission model in which the "infection" will lead to more people supporting the opinion of the infector, effectively establishing the change of the number of units in the positive, negative, and neutral parties over time. In addition, we performed extensive theoretical analysis to investigate the property of the disease-free equilibrium and to calculate the basic reproduction numbers for three different scenarios. For each system, we derive explicit solutions for the basic reproduction numbers and discuss their important implications for guidance in practice.

Keywords: Compartmental Disease Transmission Model, Susceptible-Infected-Recovered Model, Basic Reproduction Number, Rumor Transmission Model.

1. Introduction

Today, with the dramatic increase in the use of social media, communication between people has become more accessible. This results in people having easier access to all kinds of information. However, the information that has been spread is difficult to verify if it is true. People reserve various opinions and ideas about dubious details, which we call rumors [1]. Some people share their views about these rumors with others, resulting in more people having their previous thoughts changed by others with opposite ideas. As suggested by Hayakawa, it can spread on a large scale in a short time through communication chains [2]. Such transfer processes will proceed until no one continues to follow these rumors [3]. Therefore, it is essential to model the transfer of rumors to help curb the spread of rumors and eliminate the potential negative impact on society promptly and effectively.

Towards this end, compartmental disease transmission models are widely used to model state transmission in infectious diseases, using differential equations to model the change in the number of units in different states over time. For example, the Susceptible-Infected-Recovered (SIR) epidemic model assumes that when a susceptible person and an infected person come into contact, the infected person will have a probability of transmitting the disease to the sensitive person, causing the latter to become infected [4-6]. In addition, a certain percentage of infected individuals are cured over time and recovered. Recovered individuals receive lifelong immune protection from infection. Recently, variants of the SIR model have been extensively studied to model more flexible transfer mechanisms for disease or other application scenarios, e.g., the SICR model [7], the SEIR model [8], etc. However, these models cannot be effectively used for rumor transmission modeling due to the "competitive" mechanism of rumor propagation.

Specifically, for a rumor or opinion, the "infected" population can have a positive or negative thought, and since the information spread can cause "susceptible" people to be "infected" with the same positive or negative opinion corresponding to the "infector" [9]. Therefore, a critical adjustment to the rumor-based state transfer model is establishing competition among infected individuals for the direction of infection, either positive or negative.

In this paper, we propose a novel competitive infectious transfer mechanism to model the "infection" of people with positive and negative opinions to "susceptibles" or even people with opposing views. This "infection" will lead to more people supporting the idea of the infector [10]. Specifically, an infected person can hold either a positive or negative opinion about a rumor, either of which can be further refined into "opinion spreaders" and "opinion holders" [11]. A positive "opinion spreader" can disseminate information to induce "susceptibles" and opposing "opinion holders" to change their attitudes so that they have favorable opinions and vice versa. However, we assume that "opinion spreaders" with positive attitudes do not cause any change in the attitudes of "opinion spreaders" with negative attitudes because the latter are solidly opposed and can hardly be reversed by the spreader with another perspective. Thus, the proposed competitive infectious transfer model can effectively establish the change of the number of units in the positive, negative, and neutral parties over time, significantly compensating for the shortcomings of the previous SIR model and its variants in establishing opinion transfer.

In addition, we performed extensive theoretical analysis to investigate the property of the disease-free equilibrium and to calculate the basic reproduction numbers for three different scenarios. Specifically, in the first scenario, we want to control over time the "spreaders" who have a positive attitude towards the rumor. In the second scenario, given the potential harm of positive attitudes toward talks, we want to control all those who have positive attitudes toward dishes, including "spreaders" and "holders." In the third scenario, instead, we want to reduce the discussion of a word, i.e., control all "infected" people, including all "spreaders" and "holders" with positive and negative attitudes, to promote social agnosticism about the rumor. For each scenario, we derive explicit solutions for the basic reproduction numbers and discuss their important implications for guidance in practice.

2. Preliminaries

2.1. SIR Model

Compartmental disease transmission models have been extensively studied to model the number of individuals in different disease states over time. Among them, one of the most common epidemiological models is the Susceptible-Infected-Recovered (SIR) model, which divides the population into three forms: Susceptible, Infected, and Recovered, and calculates the number of people infected theoretically by a contagious 'disease' in a closed population over time.

In the SIR model, let S denote the number of susceptible individuals, I mean the number of infected individuals, R indicates the number of recovered individuals, and N represents the total population. It is assumed that when a susceptible individual comes into contact with an infected individual, the sensitive individual has a certain probability of being infected and transferring to an infected individual. Let β be the average number of contacts per person per time, multiplied by the likelihood of disease transmission between susceptible and infected individuals per contact; as shown in Figure 1, the transmission rate per time from S to I is βSI. In addition, sensitive individuals are assumed to be recovered at a constant rate λ per time, and the recovered individuals will be protected after that and cannot be re-infected.

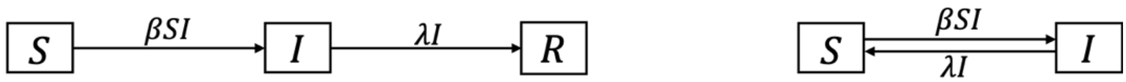

Figure 1. The flowchart of the previous SIR Model (Left) and SIS Model (Right).

Formally, the SIR model treats the change in the number of individuals in each state over time as

$$\begin{cases} \dfrac{dS}{dt} = -\beta SI \\ \dfrac{dI}{dt} = \beta SI - \lambda I \\ \dfrac{dR}{dt} = \lambda I \end{cases} \tag{1}$$

which is a nonlinear system, and the sum of the derivatives of the three states concerning time is zero, i.e.,

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0 \tag{2}$$

Let R_0 denote the primary reproduction number, which is derived as the expected number of new infections from a single disease in a population where all subjects are susceptible, i.e.

$$R_0 = \frac{\beta}{\gamma} \tag{3}$$

It is possible to do an accurate analysis of the transmission of diseases according to the, equations above. In this case, the SIR model constructed is without vital dynamics, and the model will have significant differences considering the birth rate and death rate. For both SIR models, R0 can be carried out without complex calculations, which simplifies the process of analysis, where R0 is the primary reproduction number and has practical use in determining whether an existing infectious disease can spread in a population. If $R_0 > 1$, the condition will be able to spread among people, and it is more invasive as R0 grows larger. If $R_0 < 1$, the disease will not be able to spread out, and the disease cannot invade the population.

In summary, SIR models are the fundamental techniques used to analyze disease transmission, and many models are derivatives of this basic form. In simple cases, SIR models can help discuss the infectivity of a particular disease, and their flexibility and simplicity bring convenience to the analysis of disease transmission. However, such clarity can be disadvantageous as more complicated situations and multiple negative states are discussed. For this reason, SIR models cannot analyze rumor transmission, which includes more compartments that need to be addressed and interconversion. Thus, other techniques are required to construct a rumor transmission model.

2.2. SIS Model

Another widely used transmission model is the Susceptible-Infectious-Susceptible model or SIS model. This model classifies people under a transmissible disease into two compartments: Susceptible and Infectious, which are interchangeable and examine the trend and impact of the condition indicated by the theoretical number of infections per time and in the long term.

The SIS model remains the notations in the previous SIR model, which let S denote the number of susceptible individuals at a given time, and I mean the number of infectious individuals at a given time.

DOI: 10.54254/2753-8818/28/20230389

Unlike the SIR model, the SIS model considers diseases with no long-term immunity, and the Recovered state is absent. Moreover, it assumes a probability for a susceptible individual to become infectious after contact with an infectious individual. A fixed proportion of all the connections between susceptible and infectious individuals per time can transfer the sensitive individuals to the Infectious state, denoted by p; a fixed ratio of the possible contacts between existing easy and contagious individuals at a given time can occur, characterized by r. Let $\beta = pr$, and thus βSI is the rate of transmission rate from S to I (see Figure 1) since SI is the possible number of contacts between all the existing susceptible and infectious individuals at the moment.

Furthermore, the model assumes that infectious individuals are constantly likely to be healed and become susceptible again; the likelihood is denoted by λ. Hence the transmission rate from I to S is λI (see Figure 1). The model neglects birth and death rates, so transmissions only happen between the two states, S and I, and the population is closed.

The SIS model simulates the change in the number of individuals in each compartment by the following equations

$$\begin{cases} \dfrac{dS}{dt} = \lambda I - \beta SI \\ \dfrac{dI}{dt} = \beta SI - \lambda I \end{cases} \tag{4}$$

which are the derivatives of the two states concerning time. Since the model is based on a closed population, the sum of the two derivatives is zero; that is,

$$\frac{dS}{dt} + \frac{dI}{dt} = 0 \tag{5}$$

and the sum of the number of people in the two states is a constant, *N*, that is,

$$S(t) + I(t) = N \tag{6}$$

The DFE of the model is $(N, 0)$, and the primary reproduction number of this model is

$$R_0 = \frac{N\beta}{\lambda} \tag{7}$$

This suggests that when the initial number of infectious individuals is not zero, the greater N and β are, and the smaller λ is, the more infections a contagious person can spread in the population, and thus the more unstable the DFE is. This is easy to understand: with a greater N, there are initially more susceptible individuals, so the number of possible contacts is extensive. Next, with greater β, a more significant proportion of contacts are creating new infectious individuals. Last but of least, a smaller λ indicates that infectious individuals are healed more slowly, so they will stay in an Infectious state for a longer time, infecting more people.

In conclusion, the SIS model is helpful in cases where people do not acquire long-term immunity to the disease and keep catching the illness for a period. It mainly demonstrates the competition between two compartments. However, this model does not fulfill our need to simulate the transmission of rumors. First and foremost, two competing infectious boxes transmit talks on top of the susceptible individuals, yet the SIS model only considers one contagious group. Secondly, rumor transmission includes a recovery stage, where people lose interest in the rumor and no longer mention it, so they will never be infectious or susceptible again. Still, the SIS model fails to include this. Thirdly, not all people believing in the rumor or its counterargument will be contagious because there may be weaker and unsteady believers in the society who refuse to voice their attitudes and switch from parties. In sum, the SIS model cannot successfully imitate the situations of rumor transmission, but it does provide a fundamental structure for the new model.

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2.3. SEIR Model

In many cases of the epidemic, the early period that people have already been infected but have not been infectious yet occurs. For studying these several diseases, the Susceptible-Exposed-Infected-Recovered (SEIR) model, which transforms the SIR model, is utilized, classifying the population into four states: Susceptible, Exposed, Infected, and Recovered. The number of infected individuals and the number of exposed individuals over time are counted.

Similar to the SIR model, in the SEIR model, let S denote the number of susceptible individuals, E donates the number of exposed individuals, mean the number of infected individuals, R indicates the number of recovered individuals, and N represents the total population. It is assumed that a susceptible individual has a certain probability of translating to an exposed individual after contact with an infected individual. Let β be the average number of connections per person per time, multiplied by the probability of disease transmission between susceptible and infected individuals per contact; as shown in Figure 2, the transmission rate per time from S to E is βSI. Besides, the exposed individuals be considered that there is a constant rate α to transfer to the infected individuals per time. There is also a constant rate λ to recover the infected individuals per time. The recovered individuals will be protected after that and cannot be re-infected.

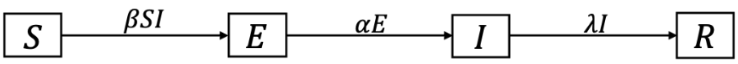

Figure 2. The flowchart of the previous SEIR Model.

Formally, the SEIR model treats the change in the number of individuals in each state as

$$\begin{cases} \frac{dS}{dt} = -\beta SI \\ \frac{dE}{dt} = \beta SI - \alpha E \\ \frac{dI}{dt} = \alpha E - \lambda I \\ \frac{dR}{dt} = \lambda I \end{cases} \tag{8}$$

which is a nonlinear system, and the sum of the derivatives of the three states concerning time is zero, i.e.,

$$\frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0 \tag{9}$$

and the sum of the populations in the three states does not change with time, i.e.,

$$S(t) + E(t) + I(t) + R(t) = N \tag{10}$$

The basic reproduction number of this model is

$$R_0 = \frac{\beta}{\gamma} \tag{11}$$

which is meaningful for the subsequent analysis. In this case, same as the analysis of the two models above, the actual birth and death have not been considered. However, during this process, SEIR models also have shown good flexibility and a basic reproduction number; R_0 can still be carried out simply. If $R_0 < 1$, the Disease-Free-Equilibrium is locally asymptotically stable; if $R_0 > 1$, it is unstable. In summary, as an extension of the SIR model, the SEIR model often applies to contagious diseases that have a latent period during the infection.

3. Proposed competitive infectious transfer model

3.1. The Proposed Model

To do theoretical analysis and investigate rumor transmission, we propose a competitive infectious model which divides the population into six states (as shown in Figure 3), S, I_+, C_+, C_-, I_-, and R. In this model, let S denote the number of susceptible individuals, I_+ denote the number of negative opinion spreaders, C_+ denote the number of negative opinion holders, C_- denote the number of positive opinion holders, I_- denote the number of positive opinion spreaders, and R represents the number of recovered individuals.

Table 1. Parameter description of the proposed competitive infectious transfer model.

Parameter	Interpretation
β_+	The average number of contacts per person per time, multiplied by the probability of rumor transmission in a connection between a "susceptible" and an individual who holds an opposing opinion
β_-	The average number of contacts per person per time, multiplied by the probability of rumor transmission in a connection between a "susceptible" and an individual who holds a favorable opinion
ε	The attenuation rate of a transformation from a negative opinion holder into a positive opinion holder and vice versa
q_+	The average probability of individuals having a negative opinion being a negative opinion spreader
q_-	The average probability of individuals having a positive opinion being a positive opinion spreader
λ_1	Recovered rate of negative opinion spreaders
λ_2	Recovered rate of negative opinion holders
λ_3	Recovered rate of positive opinion holders
λ_4	Recovered rate of positive opinion spreaders

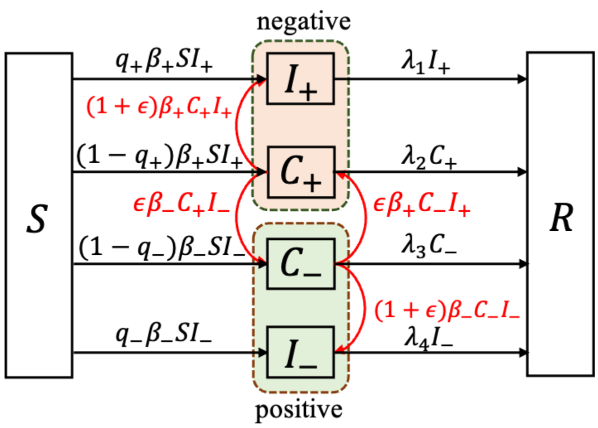

Figure 3. The proposed competitive infectious transfer model.

Applying this model, the transition between compartments can be expressed in monomials containing multiple parameters (see Table 1). And such a system can be defined by the following system of ordinary differential equations:

DOI: 10.54254/2753-8818/28/20230389

$$\begin{cases}\frac{dS}{dt} = -\beta_+ S I_+ - \beta_- S I_- \\ \frac{dI_+}{dt} = q_+ \beta_+ S I_+ + (1+\varepsilon)\beta_+ C_+ I_+ - \lambda_1 I_+ \\ \frac{dC_+}{dt} = (1-q_+)\beta_+ S I_+ + \varepsilon\beta_+ C_- I_+ - \lambda_2 C_+ - (1+\varepsilon)\beta_+ C_+ I_+ - \varepsilon\beta_- C_+ I_- \\ \frac{dC_-}{dt} = (1-q_-)\beta_- S I_- + \varepsilon\beta_- C_+ I_- - \lambda_3 C_- - (1+\varepsilon)\beta_- C_- I_- - \varepsilon\beta_+ C_- I_+ \\ \frac{dI_-}{dt} = q_- \beta_- S I_- + (1+\varepsilon)\beta_- C_- I_- - \lambda_4 I_- \\ \frac{dR}{dt} = \lambda_1 I_+ + \lambda_2 C_+ + \lambda_3 C_- + \lambda_4 I_- \end{cases} \tag{12}$$

According to this system of differential equations, the disease-free equilibrium and the reproduction number can be calculated using specific techniques. The following passage will discuss three scenarios taking distinct objects as negative states. We will determine each system's disease-free equilibrium and primary reproduction number and discuss their implications.

3.2. Theoretical Analysis Taking Negative Rumor Spreaders as Negative States

In the first case, the compartment of rumor spreaders, I_+, is regarded as negative. After plugging in $I_+ = 0$ into the system at equilibrium, one DFE is obtained: ($S(t)$, 0, 0, 0, 0, 0). Without loss of generality, let $S(t) = 1$ at the DFE so that it becomes (1, 0, 0, 0, 0, 0). It is also clear that as long as I_+ is one of the negative states, the disease-free equilibrium will always be (1, 0, 0, 0, 0, 0).

As suggested by P. van den Driessche and James Watmough [12], two vectors $\mathcal{F}$ and $\mathcal{V}$ can be constructed according to the differential equations. With I_+ being the negative state, the vectors are built such that

$$\mathcal{F} = \begin{pmatrix} q_+\beta_+ S I_+ + (1+\varepsilon)\beta_+ C_+ I_+ \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{13}$$

whose parameters, $\mathcal{F}_i(x)$, state the inflow of rumor spreaders in each compartment concerning I_+, S, C_+, C_-, I_- and R, and

$$\mathcal{V} = \begin{pmatrix} \lambda_1 I_+ \\ \beta_+ S I_+ + \beta_- S I_- \\ -(1-q_+)\beta_+ S I_+ - \varepsilon\beta_+ C_- I_+ + \lambda_2 C_+ + (1+\varepsilon)\beta_+ C_+ I_+ + \varepsilon\beta_- C_+ I_- \\ -q_-\beta_- S I_- - (1+\varepsilon)\beta_- C_- I_- + \lambda_4 I_- \\ -(1-q_-)\beta_- S I_- - \varepsilon\beta_- C_+ I_- + \lambda_3 C_- + (1+\varepsilon)\beta_- C_- I_- + \varepsilon\beta_+ C_- I_+ \\ -\lambda_1 I_+ - \lambda_2 C_+ - \lambda_3 C_- - \lambda_4 I_- \end{pmatrix} \tag{14}$$

in which the parameters say the overall outflow of individuals $\mathcal{V}_i(x)$ in each compartment, which is the individual outflow, $\mathcal{V}_{i^-}(x)$, minus the separate inflow besides the rumor spreaders, $\mathcal{V}_{i^+}(x)$, in each compartment, or $\mathcal{V}_i(x) = \mathcal{V}_{i^-}(x) - \mathcal{V}_{i^+}(x)$, and $\mathcal{F} - \mathcal{V}$ gives out the original equations.

Two matrices, F and V, are obtained by calculating the derivatives of $\mathcal{F}$ and $\mathcal{V}$, $D\mathcal{F}(x_0)$ and $D\mathcal{V}(x_0)$, concerning I_+, S, C_+, C_-, I_- and R and at the DFE (1, 0, 0, 0, 0, 0), such that $D\mathcal{F}(x_0) = \begin{pmatrix} F & 0 \\ 0 & 0 \end{pmatrix}$ and $D\mathcal{V}(x_0) = \begin{pmatrix} V & 0 \\ J_3 & J_4 \end{pmatrix}$ [12]. Calculations suggest that

$$F = q_+\beta_+ \tag{15}$$

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$$V = \lambda_1 \tag{16}$$

As suggested by P. van den Driessche and James Watmough [12], the primary reproduction number, R_0, is the spectral radius of the matrix FV^{-1}. Since F and V are 1-dimensional matrices, the calculation of R_0 is straightforward: $FV^{-1} = \frac{q_+\beta_+}{\lambda_1}$, and thus.

$$R_0 = \frac{q_+\beta_+}{\lambda_1} \tag{17}$$

This suggests that if q_+ and β_+ are more extensive, and λ_1 is smaller, then the transmission of the rumor is more likely to invade the society, and each rumor spreader will spread the word to more individuals. This is, in fact, easy to explain: when the probability of being influenced by the rumor and the proportion of the susceptible who become spreaders are high, while the recovery rate is low, there will be more rumor spreaders in society who recover slowly, and the rumor spreaders will disseminate the rumor to a lot of people, and create spreaders before themselves are transferred to the Recovered state. Therefore, the story will rage in society.

3.3. Theoretical Analysis Taking Negative Rumor Spreaders and Holders as Negative States

When the negative rumor spreaders and holders are considered as negative states (The compartments I_+ and C_+ are negative states), the DFE is (1, 0, 0, 0, 0, 0). The differential equations can state the vectors F and V.

The vector $\mathcal{F}$ elucidates the input that the individuals in the positive states transfer to the rumor spreaders and holders. The vector $\mathcal{V}$ expresses all the output of the individuals of each compartment minus the input, which is without a move from positive states to rumor negative forms. And $(\mathcal{F} - \mathcal{V})$represents the differential equations.

$$\mathcal{F} = \begin{pmatrix} q_+\beta_+SI_+ \\ (1-q_+)\beta_+SI_+ + \varepsilon\beta_+C_-I_+ \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{18}$$

$$\mathcal{V} = \begin{pmatrix} \lambda_1I_+ - (1+\varepsilon)\beta_+C_+I_+ \\ \lambda_2C_+ + (1+\varepsilon)\beta_+C_+I_+ + \varepsilon\beta_-C_+I_- \\ \beta_+SI_+ + \beta_-SI_- \\ -q_-\beta_-SI_- - (1+\varepsilon)\beta_-C_-I_- + \lambda_4I_- \\ -(1-q_-)\beta_-SI_- - \varepsilon\beta_-C_+I_- + \lambda_3C_- + (1+\varepsilon)\beta_-C_-I_- + \varepsilon\beta_+C_-I_+ \\ -\lambda_1I_+ - \lambda_2C_+ - \lambda_3C_- - \lambda_4I_- \end{pmatrix} \tag{19}$$

Hence, each element in matrix F and matrix V is defined as the partial differential equations in terms of rumor holders or rumor spreaders for each row in $\mathcal{F}$ and $\mathcal{V}$.

$$F = \begin{pmatrix} q_+\beta_+ & 0 \\ (1-q_+)\beta_+ & 0 \end{pmatrix} \tag{20}$$

$$V = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \tag{21}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{\lambda_1} & 0 \\ 0 & \frac{1}{\lambda_2} \end{pmatrix} \tag{22}$$

DOI: 10.54254/2753-8818/28/20230389

$$FV^{-1} = \begin{pmatrix} \frac{q_+\beta_+}{\lambda_1} & 0 \\ \frac{(1-q_+)\beta_+}{\lambda_1} & 0 \end{pmatrix} \tag{23}$$

Thus, the primary reproduction number R_0, which is the spectral radius of matrix FV^{-1}, is

$$R_0 = \frac{q_+\beta_+}{\lambda_1} \tag{24}$$

In conclusion, the primary reproduction number R_0 which takes rumor spreaders and holders as negative states, is identical to R0, which only takes rumor spreaders as negative states. With lower $q_+\beta_+$ and greater λ_1, the rumor diffusion is more likely to disappear. It means that the rumor tends to cease when the ratio of infected individuals to rumor spreaders decreases and the rate of recovering from rumor spreaders increases. Conversely, the story will rage.

3.4. Theoretical Analysis Taking Rumor Participants as Negative States

When I_+, I_-, C_+, and C_- are considered to be negative states, the DFE is (1, 0, 0, 0, 0, 0), which remains the same. Then we can determine the value of $\mathcal{F}$ and $\mathcal{V}$, since the original differential equations are also defined as $(\mathcal{F} - \mathcal{V})$

$$\mathcal{F} = \begin{pmatrix} 0 \\ q_+\beta_+SI_+ \\ (1-q_+)\beta_+SI_+ \\ (1-q_-)\beta_-SI_- \\ q_-\beta_-SI_- \\ 0 \end{pmatrix} \tag{25}$$

$$\mathcal{V} = \begin{pmatrix} \beta_+SI_+ + \beta_-SI_- \\ -(1+\varepsilon)\beta_+C_+I_+ + \lambda_1 I_+ \\ -\varepsilon\beta_+C_-I_+ + \lambda_2 C_+ + (1+\varepsilon)\beta_+C_+I_+ + \varepsilon\beta_-C_+I_- \\ -\varepsilon\beta_-C_+I_- + \lambda_3 C_- + (1+\varepsilon)\beta_-C_-I_- + \varepsilon\beta_+C_-I_+ \\ -(1+\varepsilon)\beta_-C_-I_- + \lambda_4 I_- \\ -\lambda_1 I_+ - \lambda_2 C_+ - \lambda_3 C_- - \lambda_4 I_- \end{pmatrix} \tag{26}$$

Applying the same method used above, we can find the two matrices, F and V, and the product of F and V^{-1}.

$$F = \begin{pmatrix} q_+\beta_+ & 0 & 0 & 0 \\ (1-q_+)\beta_+ & 0 & 0 & 0 \\ 0 & 0 & (1-q_-)\beta_- & 0 \\ 0 & 0 & q_-\beta_- & 0 \end{pmatrix} \tag{27}$$

$$V = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{pmatrix} \tag{28}$$

$$FV^{-1} = \begin{pmatrix} \frac{q_+\beta_+}{\lambda_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{(1-q_-)\beta_-}{\lambda_3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{29}$$

According to the definition of reproduction number, R_0 in the case can be derived, which is

$$R_0 = \max\left\{\frac{q_+\beta_+}{\lambda_1}, \frac{(1-q_-)\beta_-}{\lambda_3}\right\} \tag{30}$$

In this case, all rumor participants are considered negative states, meaning all "spreaders" and "holders" with positive and negative attitudes should be considered. This makes the value of R_0 to be $\frac{q_+\beta_+}{\lambda_1}$ when $\frac{q_+\beta_+}{\lambda_1} > \frac{(1-q_-)\beta_-}{\lambda_3}$, and vice versa. When $R_0 = \frac{q_+\beta_+}{\lambda_1}$, the rumor transmission is more likely to invade the society with larger q_+, larger β_+, and smaller λ_1. In this situation, a higher probability of a "susceptible" getting into contact with an individual holding an unfavorable opinion and a greater likelihood of individuals having a negative idea being a negative opinion spreader will exacerbate the rumor transmission. In addition, a lower recovered rate of negative opinion spreaders will have the same impact. When $R_0 = \frac{(1-q_-)\beta_-}{\lambda_3}$, the rumor transmission is more likely to invade the society with smaller q_-, larger β_- and smaller λ_3. In this situation, a more significant probability of a "susceptible" getting into contact with an individual holding a favorable opinion and a greater likelihood of individuals having a positive idea being a positive opinion holder will exacerbate the rumor transmission. A lower recovered rate of positive opinion holders will have the same impact.

4. Conclusion

This paper proposes a competitive infectious transmission model, in which the infection status is divided into positive and negative and considers the infection of a positive infected person to a susceptible person or even a negative infected person, also known as the "competition" mechanism. The proposed model can be effectively applied to model the spread of rumors and other real-world scenarios with infectious competition mechanisms, and more generally, it can effectively model the competitiveness of spreaders with different views or positions. In addition, extensive theoretical analyses are performed to study the disease-free equilibrium's properties and calculate the basic reproduction numbers in three other cases. For each case, we explicitly derive solutions for the basic reproduction numbers and discuss their importance for restricting the spread of rumors to show its practical guidance.

In the model constructed in this passage, infectious status is divided into positive and negative, which cannot be applied in cases where more states exist. In some cases, there may be more opinions from more than two parties spreading, influencing people, and competing for members of society. Aiming to simulate a more general competition between views such as ideologies, we can bring more infectious states into the model and extend the model for more partitions, which is left for future work.

Acknowledgement

Jiarui Men and Yongtai Hao contributed equally to this work and should be considered co-second authors.

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