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# A brief examination of the atmosphere of exoplanets

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**Abstract:** As the discovery of exoplanets increased, their characteristics are becoming a main concern for new research. Amongst the many that are discovered, some have an atmosphere of their own just like planet Earth. The purpose of this paper is to examine these atmospheres and calculate the height of the atmosphere using escape velocity. Later in this paper, the issue of the total angular momentum of the planet is examined. This momentum is separated into two different sections whose momentum is added together. The angular momentum of the solid part of the planet is calculated by considering the planet as a rigid body and applying the formula for angular momentum. The second part of calculating the angular momentum is done by using integration to determine the momentum of the atmosphere and using the height of the atmosphere to act as a limit to the definite integral. By understanding the quantities of these exoplanets, further research can be done on how these exoplanets are formed.

**Keywords:** angular momentum, height of atmosphere, exoplanets.

## 1. Introduction

Extrasolar planets are planets that exist beyond the solar system. To understand the universe observations are needed to gain a better understanding of it. Planets such as the moon are relatively close to us and scientists are able to examine samples taken. However, many planets are far away, and thus astronomers have to examine them at a distance. It is this distance that causes observation difficulties to go up exponentially. The means of gaining information are limited. Visual information made up most sources of information. Coincidentally, light travels the fastest in the universe and no medium is needed for them to propagate. Thus, understanding the properties of light is important for us to extract information from the light that is received by current equipment.

## 2. Atmosphere of exoplanets

The transit method is a good way for us to detect extrasolar planets. It mainly uses the fact that light travels in a straight line. According to [1] and [2], light from a star is partially blocked by a planet when it is passing in front of the star. By observing the star constantly, astronomers are able to determine the period of the planet by finding the time it takes for the star's brightness to dip. The transit method also brings insight into the composition of an exoplanet's atmosphere some light goes through the atmosphere of the planet. Since the discovery of the transit method, the number of discovered planets has significantly increased. According to [3] it can also be used to determine the existence of an atmosphere. In this paper, the transit method is going to be used in determining the period of a planet

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and thus, gaining insights into the motion of the planets in order for us to associate it with the autorotation rate of the planet.

The radio velocity method is able to provide a crucial parameter of the planet. The radio velocity method is built on Doppler's effect. Since planets can not produce light and thus is unable to be observed directly by using the telescope. However, when considering the planet and the star as a system, they are bound by the gravitational force that they exert on each other. Thus, by observing the movement of the star, the motion of the planet can be deduced. The wavelength of the light produced by the star is varying due to the fact that it is closing in on the observer and moving further away from the observer. Thus, the wavelength of the light will vary due to the speed of the star. This wavelength variation, explained in [4], enables us to trace back the motion of the star.

Earth has an atmosphere. It is due to this that humans are able to survive. As observations and experiments were done to better understand the atmosphere that surrounds Earth, the question of whether the atmosphere exists on other exoplanets is raised. However, the problem of detecting the atmosphere is difficult due to the nature of the atmosphere. Only light emitted can be used as a source of information to examine these atmospheres. Yet, planets are unable to produce light. In order to do research on the atmosphere of an exoplanet, light from a star has to be indented onto the planet, passing through the atmosphere. Thus, determining the composition of an exoplanet's atmosphere is difficult. Though hard, studies on the properties of the exoplanet's atmosphere are advancing. Observations such as atmospheric circulation and chemical composition like in [5] and [6] have been made over the year. These studies enable further research on how these atmospheres are formed and if the planet is suitable for other life forms.

An atmosphere is a layer of gas particles that envelopes the planet from all sides. These particles are held in place by the gravity exerted by the planet. Thus, in order for the atmosphere to exist, the gravitational force has to be strong enough for the particle to be bound by the gravitational field of the planet. Here, the assumption of the planet being spherical is made. Thus, the particles are traveling in circles around the planet. The formula for the centripetal force is applied.

$$F = m\omega^2 r = \frac{GMm}{r^2} \tag{1}$$

There will be limitations to the autorotational speed of the planet. Consequently, there will be a critical rotational period for the planet. If the rotational speed of the planet is to be exceeded then the particles in the upper atmosphere of the planet will be lost in space because the gravitational force of the planet will no longer be sufficient to hold the particle in place. However, astronomers are only able to get information from the transit method and the radial velocity method. These methods are limited for they can only determine certain aspects of the motion of the planet.

The radial velocity method tells the relation between the amplitude of the radial velocity signals and the mass of the exoplanet that has an atmosphere.

$$K = (\frac{2\pi G}{T})^{\frac{1}{3}} \cdot \frac{M_p \sin\ i}{(M_p + M_*)^{\frac{2}{3}}} \cdot \frac{1}{\sqrt{1-e^2}} \tag{2}$$

Where $M_p$ is the mass of the planet that is orbiting the star in a two-planet system. $M_*$ is the mass of the central body. The term $\sin\ i$ is here to project the mass of the planet into a desired direction. The period in the equation is given by:

$$T = 2\pi\sqrt{\frac{a^3}{GM_*}} \tag{3}$$

Where a is the major axis for the elliptical orbit of the planet. Putting (2) and (3) together gets:

$$K = (\frac{2\pi G}{T})^{\frac{1}{3}} \cdot \frac{M_p \sin\ i}{(M_p + \frac{4\pi^2 a^3}{GT^2})^{\frac{2}{3}}} \cdot \frac{1}{\sqrt{1-e^2}} \tag{4}$$

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### *2.1. Angular speed*

According to Dones and Tremaine from the Canadian Institute for Theoretical Astrophysics, University of Toronto, planetary spin can be calculated by considering the process of accretion. In their paper [7], the process of accretion is a way planets are formed. By considering how particles are attracted to the planet when the planet is spinning, they are able to examine how planets moving in a circular orbit are able to "attract" nearby particles to their body and form a planet. By examining the mechanism of how planets are formed they are able to derive a formula for the final angular speed of the planet which is given by the equation:

$$\omega = 2(\frac{m_{max}}{M}) \cdot \sqrt{\frac{GM}{R^3}} \tag{5}$$

Where $\omega$ is the final angular speed of the planet. $m_{max}$ is the mass of the largest impactor and $M$ is the final mass of the planet. The value of this ratio as explained in [8] is about 0.2. $\frac{m_{max}}{M} \approx 0.2$

Plug in this ratio to simplify equation (5):

$$\omega = 0.4 \cdot \sqrt{\frac{GM}{R^3}} \tag{6}$$

In this equation $M$ is the mass of the planet. This mass is determined using the radial velocity method's equation which is given by equation (4). The angular speed $\omega$ in the equation should be compared to the critical angular velocity of the planet which is given by equation (1). However, satisfying equation (1) only means that a particle moving in the atmosphere is not doing uniform circular motion. This does not mean that the particle can't stay in the gravitational field of the planet. Thus, the new critical condition should be:

$$\omega_{crit} R = v_{esp} \tag{7}$$

Where $v_{esp}$ in the equation should be the escape velocity for the planet. $v_{esp}$ is given by equation (8).

$$0 - \frac{1}{2} m v_{esp}^2 = -\frac{GMm}{R+h} \tag{8}$$

Where $h$ is the height of the atmosphere. Thus, $R + h$ is the distance between the particle and the center of the planet. Combining the equations (7) and (8) give:

$$\omega_{crit} = \sqrt{\frac{2GM_p}{(R+h)^3}} \tag{9}$$

Equating equation (6) with (9) gives us, note that the angular speed of the planet can not be determined directly using this formula (10):

$$0.4 \cdot \sqrt{\frac{GM}{R^3}} = \omega_{crit} = \sqrt{\frac{2GM_p}{(R+h)^3}} \tag{10}$$

In this equation $M$ is the same as $M_p$, both of them representing planetary mass. Thus, a simplified version is:

$$h \approx 1.321R$$

This is suitable for some occasions. However, using this to approximate some planets can lead to a large error. This error might be due to the fact that the planet in question is not of uniform density. Some of these errors might be due to limitations of (6). In their paper, Dones and Tremaine explicitly point out that their research is under the assumption of having a circular orbit and planets with uniform or slow-changing densities. The factor of $\frac{m_{max}}{M_p}$ can affect the result of the calculation significantly. Thus, substitute the above result with the factor $\frac{m_{max}}{M_p}$ gets equation (11):

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$$h \approx \left(\frac{0.794}{\left(\frac{m_{max}}{M_p}\right)^{0.667}} - 1\right) \cdot R \tag{11}$$

### 2.2. Angular momentum

Equation (11) is a move general case. As for $\omega_{crit}$, combine equation (4) with equation (6) can obtain the result. While $\omega_{crit}$ can be determined using the radial velocity method. However, the height of the atmosphere is hard to be defined because the transition is too subtle. The angular momentum of the planet can also be determined. The angular momentum of an object is obtained by determining the product of the rotation inertia and the angular speed of an object.

$$L = I \cdot \omega \tag{12}$$

By using the general formula for the rotational inertia.

$$I = k \cdot MR^2 \tag{13}$$

Where $k$ is a constant to be determined. By combining these two equations obtained:

$$L = k \cdot MR^2 \cdot \omega \tag{14}$$

By inserting the more general case of equation (6) to determine the angular momentum for a planet give:

$$L = k \cdot MR^2 \cdot 2 \cdot \frac{m_{max}}{M_p} \cdot \sqrt{\frac{GM_p}{R^3}} \tag{15}$$

After doing a little simplification equation (16) is obtained:

$$L = 2k \cdot \frac{m_{max}}{M_p} G^{\frac{1}{2}} M^{\frac{3}{2}} R^{\frac{1}{2}} \tag{16}$$

The $k$ in this formula is an empirical value and the ratio of the largest impactor and the final planetary mass is about 0.2. Planetary mass $M$ can be determined using the radial velocity method. From equation (4) plug in all the observed values to give the mass of the planet. Thus, obtaining the value for the angular momentum of the planet $L$. According to Karatekin [9] who is one of the authors of the paper discussing the angular momentum of atmospheres, the atmosphere will exert a frictional torque on the solid part of the planet. In this paper, he talks about how the net angular momentum of the atmosphere and the solid part of the planet's angular momentum is conserved. The angular momentum of the solid part is calculated using (16). In his paper, he also talks about how the period of the day is changing because of the frictional torque exerted on the solid part.

$$\vec{H} = \int \vec{r} \times \rho \vec{v} dV + \int \vec{r} \times \rho(\vec{\Omega} \times \vec{r}) dV \tag{17}$$

Where $\vec{r}$ is the position vector pointing away radially from the center of mass of the planet. Under the assumption of uniform density and spherical shape, the position vector will be pointing away from the geometric center of the planet. $\vec{H}$ is the angular momentum of the entire atmosphere which is assumed to be fluid, having its properties. Where $\rho$ is the density of the atmosphere. $\vec{\Omega}$ is the angular velocity of the planet. The first term is expressing the angular momentum of the atmosphere caused by the difference between the speed of the solid part of the planet and the atmosphere. The second term is representing the angular momentum of the atmosphere when observed in the frame of the solid part of the planet. However, in this paper, there is relative speed between the atmosphere and the planet. Thus, assume that the angular speed of the atmosphere relative to the planet to be $\omega_{rel}$. The new equation that describes the relationship between the speed of the particle and radius is given by:

$$(\omega_{rel} + \omega)R = v_{esp} \tag{18}$$

Equation (16) should be unchanged because the angular momentum of the solid part of the planet is not affected by $\omega_{rel}$.

$$L = 2k \cdot \frac{m_{max}}{M_p} G^{\frac{1}{2}} M^{\frac{3}{2}} R^{\frac{1}{2}}$$

$L$ is the angular momentum of the planet around the polar axis. This momentum should be equal to the momentum of the atmosphere plus the momentum of the atmosphere. Thus, the total angular momentum of the planet is given by:

$$L = 2k \cdot \frac{m_{max}}{M_p} G^{\frac{1}{2}} M^{\frac{3}{2}} R^{\frac{1}{2}} + \int_{R}^{R+h} \rho(r) \cdot (\omega + \omega_{rel}) r^2 dr \tag{19}$$

Where $\rho(r)$ is the density of the atmosphere as a function of the distance from the center of the planet. $\omega$ is the rotational speed of the planet with respect to the polar axis. Thus, combining equation (17) with the equation (5):

$$L = 2k \cdot \frac{m_{max}}{M} G^{\frac{1}{2}} M^{\frac{3}{2}} R^{\frac{1}{2}} + \int_{R}^{R+h} \rho(r) \cdot [2(\frac{m_{max}}{M}) \cdot \sqrt{\frac{GM}{R^3}} + \omega_{rel}] r^2 dr \tag{20}$$

## 3. Conclusion

Although the critical speed for a particle in the atmosphere of a planet can be determined. However, this is not a very accurate result. Due to general atmospheric circulation, the speed at which the atmosphere of a planet is moving can vary from the speed at which planets are rotating. Venus's atmosphere, in fact, is moving at a much faster rate than its rotating speed. Thus, there will be limitations for this method because this method is built under the assumption of particles being relatively still when compared to the planet. Further research can focus on the situation where the planet's atmosphere's moving speed is different from that of the planet's rotating one. There are currently papers like [10] that focus on the issue of the atmosphere's circulation movement, for example, several methods for calculating atmospheric movement for the different kinds of planets are explained in Showmen's paper, however, it is yet to be put together with the prerequisites for forming atmosphere to form a more accurate result. Despite these limitations, the angular momentum of the planet and height of the atmosphere of the planet can still be calculated by using the right parameters. The results from these calculations can be used in future studies can enable further explorations of the exoplanets.

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