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The relationship between N_{coll} and eccentricities based on Glauber model

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Abstract. This report aims to determine the probability range of the number of collisions N_{coll} under the optimum collision condition. The optimum condition was obtained by determining the average values of the given variables in the Glauber Model. Graphs of N_{coll} against eccentricity 1, 2, and 3 (Ecc1, 2, and 3) were plotted respectively according to the data generated from the Glauber Model Simulation in CERN ROOT. Then, the optimum average value was plotted as a vertical line on the graph to determine the range of N_{coll}. The probability ratio of an optimum collision versus maximum probability in an event was concluded to fall in a certain range of 0.30 ± 0.07, and this range is verified to be a stable range that could be used for prediction of optimum collision numbers in future nuclei collision experiment. This probability ratio can be used to predict the optimum collision with only N_{coll} provided.

Keywords: Optimum Collision Condition, Eccentricity, Number Of Collisions.

1. Introduction

This report deals with the correlation between the probability distribution of collision events and the differences between the long and short axes of the collision area. Moreover, determining the optimum range is closely related to optimizing collision events between nuclei. The first possible application is to predict the optimum range using only one variable N_{coll}. In an actual experiment, it was difficult to obtain the optimum collision. This is because the initial state of the nuclei is difficult to obtain after the collision, which means that the lengths of the longest and shortest paths are difficult to determine when using LHC at CERN. The second is to determine the condition that satisfies the optimum collision, and then apply the condition determined to future nuclei collision experiments. In this simulation, the Monte Carlo method, Glauber simulation, and Pb-Pb collision are selected; thus, the initial conditions can be stored by the algorithm for the further calculation to determine the optimum collision; heavy colliding nuclei pass through and interact with the medium. The collisions between Pb and Pb particles are sometimes asymmetric owing to inelastic collisions, which means that kinetic energy loss occurs according to Mehndiratta A. et al. To select and predict the best collision in a collision event at a certain probability distribution, we must determine the specific relationship between N_{coll} and eccentricities. Figure 1, adapted from Adrian Dumitru and Yasushi Nara's paper, indicates an exponential relation.

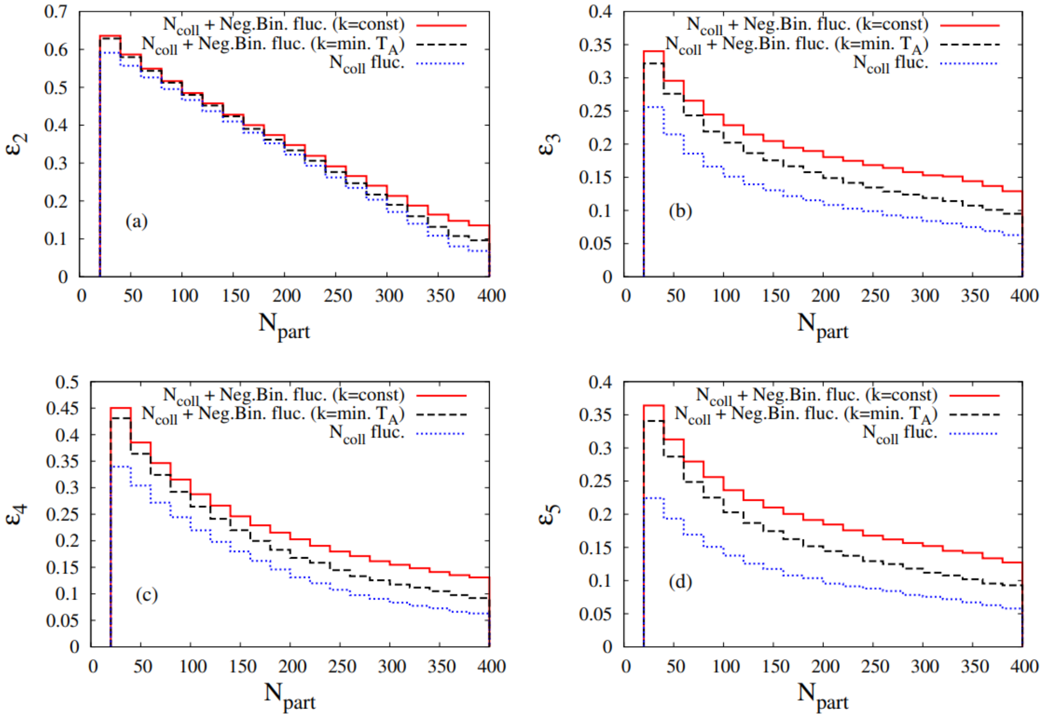

Figure 1. The diagram above shows the relationships between eccentricities (Ecc1, 2, 3, and 4) and the number of collision-participating particles. Since N_{coll} is the number of collisions, i.e., particle pairs that collide together, there potentially exists an exponential relationship similar to this graph presented [1].

From Figure 1, Pb+Pb collision data was obtained. Graphs were generated by plotting eccentricities against the number of participants N_{part}. From these graphs, the exponential relationship could be identified. Since N_{part} and N_{coll} are the same types of variables, it is interesting to explore the relationships between N_{coll} and eccentricities as well. The method of this experiment and raw data with Glauber Simulation will be presented below.

2. Methods

The experiment was based on the Glauber Model Simulation. ROOT from CERN was used to run a package called runglauber_v3.2. The Glauber Model indicates the correlation between quantum fluctuations and density. In the computer simulation, the collision event is randomized by Monte Carlo Simulation, a mathematical simulation that provides a random impact parameter b within a specific range to generate a stable result. These are presented in the TGlauberMC. After installing ROOT on the computer and loading the shared library, we used runAndSaveNtuple(5000), where the number in brackets represents the number of events the TGlauberMC generated using the Monte Carlo Method.

The nuclei for collision are Pb-Pb; some basic constant values are given below: For Pb-Pb collisions, Pb atoms have a mass of 207u, $\sqrt{sNN} = 343\ GeV$. The nuclear charge density is based on the formula of Fermi Distribution below, the value of R, a, and ω for Pb are 6.62, 0.546, and 0 respectively [2].

$$\rho(r) = \rho_0 \frac{1 + \omega(r - R)^2}{1 + e^{\frac{r-R}{a}}} \tag{1}$$

The exponential term in the denominator shows that an exponential relationship may be identified in the final plots.

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The N_{coll} will be a range as the graph may not be flat with all values at that point equal to the same N_{coll}. After several comparisons, the range of N_{coll} obtained was combined to determine the optimum N_{coll} range. I first came up with a N_{coll} value for the same number of events as 100,000; then I moved to different amounts of data, such as 5,000, 10,000, and further. The final step was plotting N_{coll} against the average eccentricities to obtain a certain relationship to predict the optimum N_{coll} range for each collision. The text file generated from ROOT was extracted and put into Python code to calculate the average values of eccentricities 1, 2, and 3 (Ecc1, 2, and 3).

Since the graphs of N_{coll} versus eccentricities have general patterns like Fig.1 which shows an exponential relationship, a code to extract the boundary data was designed to reach the effect in Fig. 1. The basic logic is:

1) Extract Ecc1, Ecc2, Ecc3, and N_{coll} from the data file generated in ROOT and use a two-dimensional array to store each pair of eccentricity and N_{coll} values.

2) Group all duplets of data, under the criterion of grouping duplets with the same N_{coll} value in descending order.

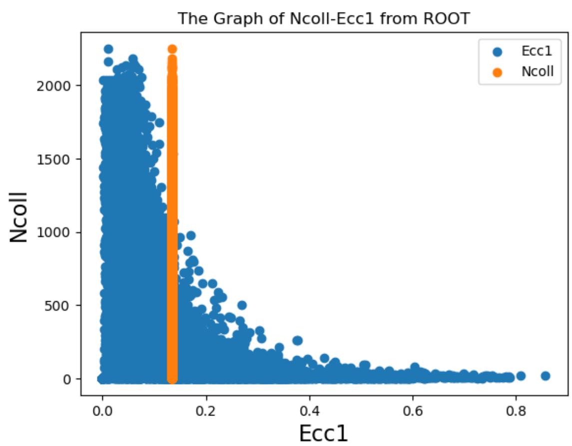
3) Extract the maximum data for each value of N_{coll}. In this step, the initial expectation is to use the first ten data but later the graph plotted is considered too close to the original. The accuracy of curve fitting will be affected.

4) Plot all these points selected on one diagram, and use curve_fit in Scipy to do exponential function fitting, reading the parameters, a, b, and c where the program generates

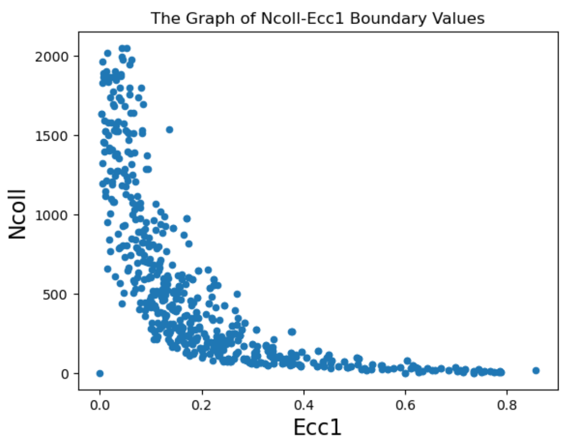
5) Plot the fitted curve using plotting software. Then, the line x = average eccentricity is plotted to obtain the intersection. The N_{coll}value of intersections is read to obtain the optimum N_{coll} value.

6) Ratio of optimum N_{coll} and maximum N_{coll} is calculated to find the percentage range of optimum.

3. Raw Data and Graphs



(a)



(b)

Figure 2. This graph uses 10000 collision events and Ecc1 values as an example. (a) The original graph of $N_{coll} - Ecc1$ was generated from .csv files obtained from CERN ROOT. (b) Using written decimation code to extract the maximum value of eccentricity at each N_{coll} value; the boundary was formed shown in the figure. With this boundary, curve fitting was done.

Figure 2 gives an example of data extraction. The original N_{coll} versus Ecc1 graph was plotted based on the unextracted dataset. Then, input the data set into the algorithm. The maximum eccentricity value of each N_{coll} was selected and stored in a data frame of two columns, the first one is 'Ecc1', and the second one is 'N_{coll}'. This data frame is plotted in Figure. 2(b). Now what we need to determine is the N_{coll} value of the line of average eccentricity intersecting with the boundary of the dataset. To make the data more accurate, Geogebra® is used for fitted curve plotting, shown in Fig. 3:

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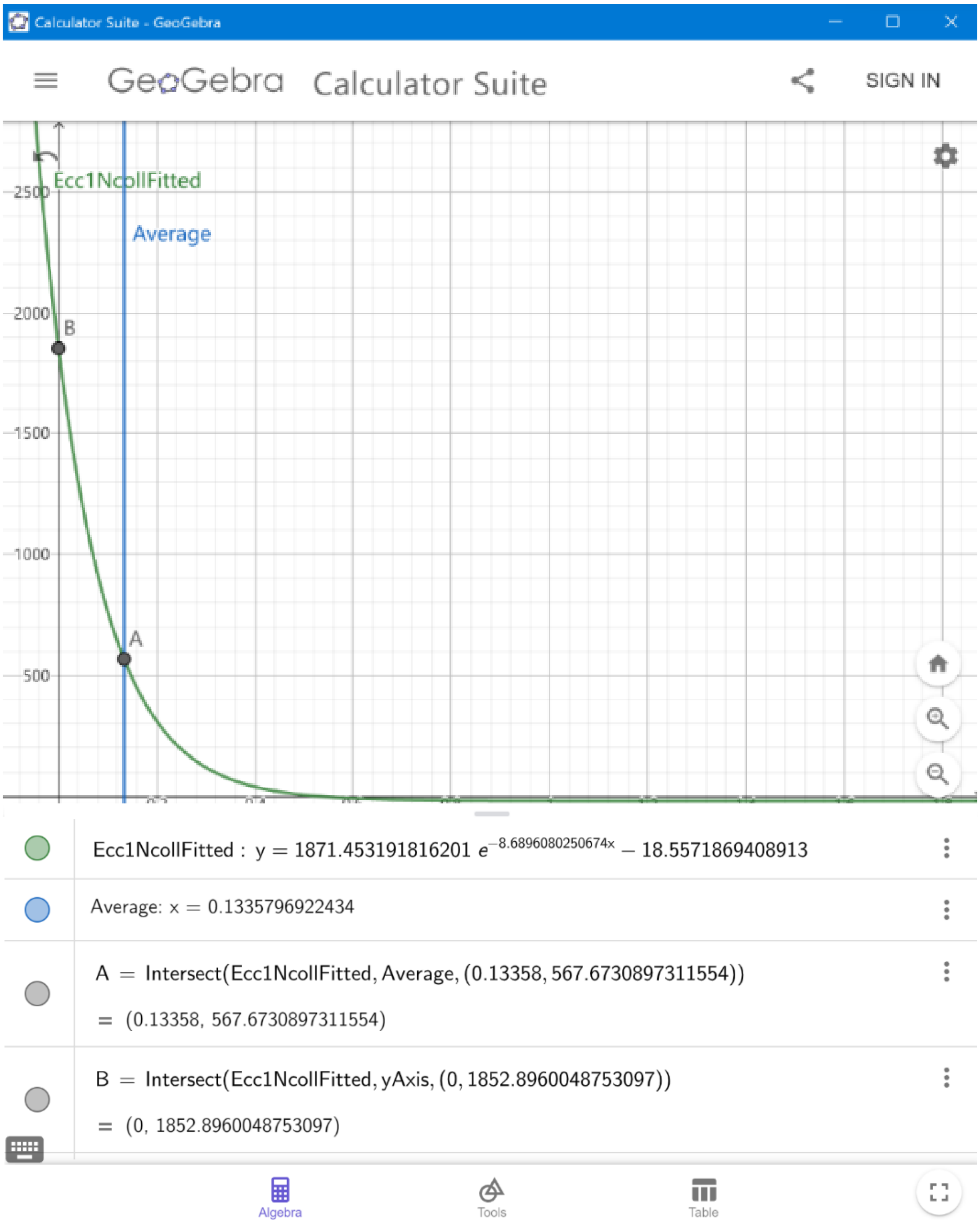

Figure 3. Fitted curves with known parameters, $y = ae^{bx} + c$ were plotted using Geogebra® (green curves), and with $x = average\ eccentricity$ (blue line).

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Table 1. These values are obtained in Geogebra® plotting software. The Max Interception is where the $Ecc = 0$, and the curve intersect with the vertical axis. The optimum intersection is the y-value of the intersection between the fitted curve and line $x = average\ eccentricity$

collision events	eccentricity used	average eccentricity	a_fit	b_fit	c_fit	Max Interception	Optimum Interception	Opt/Max Ratio	Average Percent	Uncertainty
10000	Ecc1	0.134	1871.453	-8.690	-18.557	1852.896	567.675	0.306	0.300	0.072
	Ecc2	0.435	2255.021	-2.841	-193.852	2061.169	461.582	0.224		
	Ecc3	0.271	2169.746	-3.117	-208.147	1961.599	722.803	0.368		
50000	Ecc1	0.135	2500.673	-8.411	-7.410	2493.263	796.002	0.319	0.332	0.071
	Ecc2	0.441	2728.587	-2.218	-404.236	2324.351	622.655	0.268		
	Ecc3	0.275	2868.997	-2.592	-389.953	2479.044	1015.638	0.410		
100000	Ecc1	0.135	2725.588	-8.260	2.925	2728.513	895.211	0.328	0.347	0.069
	Ecc2	0.435	2955.042	-2.083	-486.791	2468.251	707.929	0.287		
	Ecc3	0.276	3139.376	-2.442	-461.830	2677.547	1139.673	0.426		

Table 1 presents the parameters of exponential functions fitted from the deduced boundary datasets. The data obtained from the graph shows that the optimum range is represented in terms of uncertainties for Pb-Pb collision. This range is mainly used in the prediction of further collisions. The N_{coll} value has a fixed range from the data collected in 10000, 50000, and 100000 collision events.

4. Discussion

From the data above, we could find that the ratio of Optimum versus mass for different numbers of collision events becomes stable with the range of 0.30 ± 0.07. This means that the exponential relation exists with a stable result of prediction range, where the Optimum collision occurs at 0.30 of the Maximum collision numbers, with a total percentage magnitude of 0.14. This percentage range is relatively large due to the intrinsic nature of randomness in nuclei collisions. The ratio is stable at 0.30; however, more extensions should be made to reduce the fluctuation range. For example, eccentricity 4 and 5 could also be introduced to check the ratio, and other variables like N_{part}, and impact parameter b can be also used to verify the conclusion. Nuclei collision is a random process, but we could find out a stable exponential relationship between eccentricities and collision numbers and a stable optimum versus maximum collision ratio [3].

5. Conclusion

This experiment deals with the determination of a stable optimum percentage range and the exponential relationship between eccentricities and the number of collisions, inspired by the discovery of the exponential relation between participant numbers and eccentricities. The result obtained is the optimum collision mostly falls in 0.30 ± 0.07 times the maximum collision numbers in an event. The exponential relationship is established so this range could be used to predict the number of optimum collisions reliably with only Max collision provided. Moreover, we could also use these ranges to check the initial conditions that collisions happened in this range to make the ratio larger to apply these discovered initial conditions in the future experiment to yield more complete and symmetric collisions to avoid the energy loss mentioned in the introduction. In all, verifying the exponential relationship between eccentricity and collision numbers and determining the optimum collision percentage range can be applied in the upcoming predictions and help scientists identify the optimum collisions.

The limitation presented in the experiment is:

1) More sets of data should be used. Collision events at numbers of 50000 and 100000 could be tried to obtain more general predictions since they have larger sample space.

2) Other kinds of collision could be considered. For example, Au-Au collision, proton-proton collision, and proton-alpha particle collision. Different types of events will lead to different optimum collision ranges.

3) Different types of functional relationships could be tried to determine the best equation for curve fitting. Exponential relations may be only one of them.

The extension can be done as follows:

As B. Alver, et al mentioned in their paper, elliptical flow fluctuations in RHIC can be comparatively significant from Cu+Cu collision to other collisions. Even data in Cu-Cu collisions could vary to the level of Au-Au collisions [4], and much larger than expected from hydrodynamical models [5], the main extension I would like to take on is to conduct different types of collision involves different kinds of particles. Then, I could predict each kind of particle’s optimum collision range and conclude an equation or functional relation between relative particle mass and the optimum collision numbers.

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