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Optimizing the counter service based on linear programming and queue theory

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Abstract. With the progress of the social economy, the pace of people's life and work is accelerating, and queuing has gradually become a key factor affecting the service level and quality of enterprises. Although the queuing problem has been gradually optimized, there are still some problems such as queue congestion and confusion, inadequate work of staff, and low efficiency. It can be seen that effective measures have been taken to properly solve the queuing problem in China's service industry and other industries. This paper is aimed at using the method that combined linear programming with queue theory to develop 3 steps to optimize the manpower scheduling problem for a real-world situation. A case study of the local government service counter is provided. After applying our model to the case problem, we have achieved three goals: to reduce the number of staff to a more accurate state. Our research finds that there exist some real problems with the service like their service rate is a bit low and needs to improve. In addition, this model has application value for a wide range of other fields, such as banks, post offices and other scenarios.

Keywords: Linear programming, Queue theory, Service.

1. Introduction

Queuing has become a very normal phenomenon in our daily life. Everyone hopes that we will have a better queue system. According to a survey of clients' feedback about services provided by the government, the long waiting time has been the issue that gets the most attention [1]. "We have to wait in line for two hours for something that can be done in five minutes." A woman living in the major city in Southern China, Shenzhen, said. "Waiting is frustrating, demoralizing, agonizing, aggravating, annoying, time-consuming and incredibly expensive," an advertisement for Federal Express said. Nowadays, the question of how to provide high-efficiency service and low waiting time for the customer is a necessary topic for service industry providers to think about [2].

In 2012, the Chinese government initiated a reform of the administrative system, which is aimed at improving its efficiency of the administrative system [3]. At the same time, to cut costs, the government has introduced administrative reforms that have reduced the number of staff at service counters, but the problem of queues during rush hours may arise. This paper will use the linear programming model which applied to queue theory to enhance the efficiency of counter service in the local government service hall. Queueing theory is the mathematical study of waiting lines or queues; and is generally considered a branch of operations research. Erlang conducted his initial investigation of it in 1909 with the intention of lessening telephone exchange congestion [4]. Based on his research, he solved M/D/1 queue and

M/D/k queue model. Further studies were conducted by more people. Pollaczek and Khinchin developed the Pollaczek-Khinchine formulas based on M/G/1 model [5]. Currently, queue theory is used in computing, telecommunication, traffic engineering [6], and, namely in industrial engineering, in the design of factories, offices and so on.

With the development of advanced management techniques, the method of manpower scheduling is a critical problem for companies to improve. “Manpower scheduling is the process by which the daily manpower level for each craft or skill is selected to complete the work in the most efficient (orderly, economical, safe) manner [7].” Scheduling issues were divided into three categories by Morris and Showalter: Shift scheduling, days-off scheduling, and tour scheduling [8]. Khoong, C.M. classify the staffing schedule as steady state manpower flows and multi-period staff movements [9]. There are also some research papers using the classification method like attributes of different industries to deal with different problems in different companies.

Nowadays, in government services, queue systems controlled by ticket technology are frequently used [10]. When users interact with the system, they receive a number as feedback that indicates how many clients are currently online. Users then choose whether to join the queue. If they choose to utilize this system, the system will give them a ticket and they will get in the queue.

The workforce scheduling issue in the past was based on experience, however, this approach has several disadvantages when used in practice [11]. One is that we may make some mistakes and expend time. In this passage, this paper will put forward a new scheduling system using the method of linear programming combined with queuing theory. My research goal for this problem is trying to improve the working efficiency of the workers while ensuring to reduce in the clients’ waiting time.

The structure of writing this paper is an exploration process from theory to application. Firstly, this paper will construct the models for the analysis of the queue system using the linear programming method with defined variable notations and assumptions. Next step, this paper will fill this model using the data gathered from one local government service center in China and present numerical results for this real case. Besides, I will do a sensitivity analysis of certain parameters. In the end, the conclusion and improvement suggestions will be addressed.

2. Methodology

2.1. Model-based on queuing theory and linear programming

To solve the real problem, this study will use queuing theory and linear programming. In order to build the model, this paper first defines the notations:

v_l number of employees in shift type l

λ_t average client arrival rate throughout time slot t

ρ_t average server’s utilization rate at time slot t

μ mean service rate for server

m number of all employees

This paper has some assumptions for the model:

1) The efficiency of the service is the same and all employees can handle all the customer needs.

2) Only one customer will be served at a time at each counter.

3) We assume that the interarrival-time distribution of customers is the exponential distribution.

4) The queuing discipline is first come-first served.

In this paper, two models will be formulated based on the two working time slot tables. One is the weekday working time slot and the other is the working time slot for Saturday and Sunday. The office hours are also different. The main goal for these two models is tried to minimize daily manpower demand, which is in line with the country's goal of streamlining the administrative system and improving work efficiency. Based on what we find in Model A and B, Model C tries to make the manpower scheduling more reasonable and tries to make counters’ monthly working days to a minimum.

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2.1.1. Daily Minimum Demand of Manpower. The model of service counter conforms to the m/m/n model in queuing theory. Since $\rho = \frac{\lambda}{m\mu}$, we have $m = \frac{\lambda}{\rho\mu}$, which is the right side of the inequalities. In the real scenario, all employees do not have the same work schedule. Our model construction takes the smallest time slot as the analysis unit, and the time of time slot is 30 minutes, which is basically the same as the scheduling in real life. The analytical logic of this paper is as follows: Firstly, the different scheduling time slot tables of all employees in the service center were counted. Then we can conclude the same schedule type according to the schedule between them. The value of variable X is the number of employees who have the same type of schedule. In terms of the variables for each time slot, calculate the value on the right side of the inequality, that is, the number of employees needed in this time slot. The left-hand side of this inequality is the total number of employees working in this time slot. Our goal is to reduce the total number of days employees required.

Minimize

$$Z = \sum_{i=1}^{m} v_i \tag{1}$$

Subject to

$$\sum v_i(i = exist\ shift\ type\ l\ in\ t\ time\ slot) \geq \frac{\lambda_t}{\mu\rho_t} \tag{2}$$

$$v_1, \dots, v_n \geq 0 \tag{3}$$

According to China government regulations, employees must work five days a week. The service center is open from 8:00 until 17:30 on weekdays and from 8:30 until 12:00 on weekends. The time for a lunch break is from 11:30 to 14:00 from Monday to Friday, only a few workers are on demand and the number of them is about 5-6 since most of them need to have lunch breaks for this time slot. This approach aims to reduce staff while still meeting the demand for each time slot. The operating schedule for counters from Monday through Friday is shown in Table 1. Then, the different shift types of them are shown in Table 2.

Minimize

$$Z = v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7 + v_8 \tag{4}$$

Subject to

$$v_2 + v_3 + v_4 + v_5 + v_7 + v_8 \geq \frac{\lambda_1}{\mu\rho_1} \tag{5}$$

$$v_2 + v_3 + v_4 + v_5 + v_6 + v_7 + v_8 \geq \frac{\lambda_2}{\mu\rho_2} \tag{6}$$

$$v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7 + v_8 \geq \frac{\lambda_t}{\mu\rho_t} \tag{7}$$

$$t = 3,4,5,6,17$$

$$v_1 + v_2 + v_3 + v_4 + v_6 \geq \frac{\lambda_7}{\mu\rho_7} \tag{8}$$

$$v_1 + v_3 + v_6 \geq \frac{\lambda_t}{\mu\rho_t}, t = 8,9 \tag{9}$$

$$v_2 + v_4 + v_5 + v_7 + v_8 \geq \frac{\lambda_t}{\mu\rho_t}, t = 10,11 \tag{10}$$

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$$v_1 + v_4 + v_5 + v_7 + v_8 \geq \frac{\lambda_{12}}{\mu\rho_{12}}, t = 10,11 \tag{11}$$

$$v_1 + v_4 + v_5 + v_6 + v_7 + v_8 \geq \frac{\lambda_{13}}{\mu\rho_{13}} \tag{12}$$

$$v_1 + v_2 + v_3 + v_5 + v_6 + v_7 + v_8 \geq \frac{\lambda_t}{\mu\rho_t} \tag{13}$$

$$t = 14,15$$

$$v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_8 \geq \frac{\lambda_{16}}{\mu\rho_{16}} \tag{14}$$

$$v_1 + v_2 + v_3 + v_4 + v_6 + v_7 + v_8 \geq \frac{\lambda_{18}}{\mu\rho_{18}} \tag{15}$$

$$v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7 \geq \frac{\lambda_{19}}{\mu\rho_{19}} \tag{16}$$

$$v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8 \geq 0 \tag{17}$$

The objective function is to try to minimize the daily demand for manpower, and I set the constraints (5)-(16) to aim at meeting the requirement for personnel during each time slot. The last one just keeps all the variables are not negative.

The equations for model B are for the work schedules for Saturday and Sunday (Table 3). Model B's objective function is the same as Model A's. The distinction is that the time slot less than Model A.

Subject to

$$v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7 + v_8 \geq \frac{\lambda_t}{\mu\rho_t}, \tag{18}$$

$$t = 2,3,4,5,6,7,8$$

$$v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8 \geq 0 \tag{19}$$

The target function was to ensure to meet the requirement for personnel during each time slot. One non-negativity constraint is also present.

2.2. *Extension model based on queuing theory and linear programming*

2.2.1. Optimal working days schedule. In this model, more variables will be added:

i ith employee; $i \in m$

p working days within a month

k k th day in a month; $k \in p$

t t th time slot in a working day; $t \in w$

q number of shift types in a day

l lth shift type in a day; $l \in q$

b_{kt} number of working employees at time slot t on day k

u_{ikl} indicator of employee i assigned to shift type l on day k

if $u_{ikl} = \begin{cases} 1, employee\ i\ need\ to\ work, \\ 0, employee\ i\ do\ not\ need\ to\ work. \end{cases}$

a sum of working days for each employee in one month

The data of variables for model C will be partially from Model A and B.

To simplify our model, assume only 2 shift pattern exists and only 3 employees in the company

Minimize

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$$a, \tag{20}$$

Subject to

$$\sum_{k=1}^{p}\sum_{l=1}^{q} u_{ikl} = a\,, \forall i \epsilon m, \tag{21}$$

$$\sum_{k=1}^{p}\sum_{l=1}^{q} a \ \leq 22\,, \forall i \epsilon m, \tag{22}$$

$$\sum_{i=1}^{m} (u_{ik2}) \geq b_{k1}, \forall k \in D, \tag{23}$$

$$\sum_{i=1}^{m} (u_{ik2}) \geq b_{k2}, \forall k \in D, \tag{24}$$

$$\sum_{i=1}^{m} (u_{ik1} + u_{ik2}) \geq b_{kt}, \forall k \in D \ and \ t = 3,4,5,6,17, \tag{25}$$

$$\sum_{i=1}^{m} (u_{ik1} + u_{ik2}) \geq b_{k7}, \forall k \in D, \tag{26}$$

$$\sum_{i=1}^{m} (u_{ik1}) \geq b_{kt}, \forall k \in D \ and \ t = 8,9, \tag{27}$$

$$\sum_{i=1}^{m} (u_{ik2}) \geq b_{kt}, \forall k \in D \ and \ t = 10,11, \tag{28}$$

$$\sum_{i=1}^{m} (u_{ik1}) \geq b_{k12}, \forall k \in D, \tag{29}$$

$$\sum_{i=1}^{m} (u_{ik1} + u_{ik2}) \geq b_{k13}, \forall k \in D, \tag{30}$$

$$\sum_{i=1}^{m} (u_{ik1} + u_{ik2}) \geq b_{kt}, \forall k \in D \ and \ t = 14,15, \tag{31}$$

$$\sum_{i=1}^{m} (u_{ik1} + u_{ik2}) \geq b_{k16}, \forall k \in D, \tag{32}$$

$$\sum_{i=1}^{m} (u_{ik1} + u_{ik2}) \geq b_{k18}, \forall k \in D, \tag{33}$$

$$\sum_{i=1}^{m} (u_{ik1} + u_{ik2}) \geq b_{k19}, \forall k \in D, \tag{34}$$

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$$\sum_{i=1}^{m} \sum_{k=7j-1}^{7j} (u_{ik1} + u_{ik2}) \geq b_{kt}\,, \tag{35}$$

$$t = 2, \dots, 8 \; and \; j = 1, \dots, 4,$$

$$\sum_{l=1}^{q} u_{ikl} \leq 1, \forall i \in m \; and \; \forall k \in p, \tag{36}$$

$$Y_{ikl} = 1 \; or \; 0, \forall i \in m, \forall k \in p \; and \; \forall l \in q, \tag{37}$$

The objective function tries to minimize the total number of working days for each employee in one month; Constraint (21) indicates every worker will have the same number of working days within a month; Constraint (22) means all staff will work at most 22 days per month. Constraints (23)-(34) show that must be sufficient employees assigned to each time slot on weekdays; Constraint (35) demonstrates that there must be sufficient employees assigned to each time slot on weekends; For constraint (36), it makes sure that the number of assigned shift type for one staff is one; Constraint (37) is integer constraint.

Table 1. Time Slot.

	Time Slot	Counter 1	Counter 2	Counter 3	Counter 4	Counter 5	Counter 6	Counter 7	Counter 8	Counter 9	Counter 10
1	8:30-9:00	-	▪	▪	-	▪	▪	-	▪	-	▪
2	9:01-9:30	-	▪	▪	-	▪	▪	▪	▪	▪	▪
3	9:31-10:00	▪	▪	▪	▪	▪	▪	▪	▪	▪	▪
4	10:01-10:30	▪	▪	▪	▪	▪	▪	▪	▪	▪	▪
5	10:31-11:00	▪	▪	▪	▪	▪	▪	▪	▪	▪	▪
6	11:01-11:30	▪	▪	▪	▪	▪	▪	▪	▪	▪	▪
7	11:31-12:00	▪	▪	▪	▪	▪	▫	▪	▫	▪	▫
8	12:01-12:30	▪	▫	▪	▪	▫	▫	▪	▫	▪	▫
9	12:31-13:00	▪	▫	▪	▪	▫	▫	▪	▫	▪	▫
10	13:01-13:30	▫	▪	▫	▫	▪	▪	▫	▪	▫	▪
11	13:31-14:00	▫	▪	▫	▫	▪	▪	▫	▪	▫	▪
12	14:01-14:30	▪	◦	▫	▪	▪	▪	▫	▪	▫	▪
13	14:31-15:00	▪	◦	▫	▪	▪	▪	▪	▪	▪	▪
14	15:01-15:30	▪	▪	▪	▪	◦	▪	▪	▪	▪	▪
15	15:31-16:00	▪	▪	▪	▪	◦	▪	▪	▪	▪	▪
16	16:01-16:30	▪	▪	▪	▪	▪	▪	▪	◦	▪	▪
17	16:31-17:00	▪	▪	▪	▪	▪	▪	▪	▪	▪	▪
18	17:01-17:30	▪	▪	▪	▪	▪	◦	▪	▪	▪	▪
19	17:31-18:00	▪	▪	▪	▪	▪	▪	▪	▪	▪	-

Table 2. Shift Type.

	Time Slot	Shift 1	Shift 2	Shift 3	Shift 4	Shift 5	Shift 6	Shift 7	Shift 8
1	8:30-9:00	-	▪	▪	▪	▪	-	▪	▪
2	9:01-9:30	-	▪	▪	▪	▪	▪	▪	▪
3	9:31-10:00	▪	▪	▪	▪	▪	▪	▪	▪
4	10:01-10:30	▪	▪	▪	▪	▪	▪	▪	▪
5	10:31-11:00	▪	▪	▪	▪	▪	▪	▪	▪
6	11:01-11:30	▪	▪	▪	▪	▪	▪	▪	▪
7	11:31-12:00	▪	▪	▪	▪	▫	▪	▫	▫
8	12:01-12:30	▪	▫	▪	▫	▫	▪	▫	▫
9	12:31-13:00	▪	▫	▪	▫	▫	▪	▫	▫
10	13:01-13:30	▫	▪	▫	▪	▪	▫	▪	▪
11	13:31-14:00	▫	▪	▫	▪	▪	▫	▪	▪
12	14:01-14:30	▪	◦	▫	▪	▪	▫	▪	▪
13	14:31-15:00	▪	◦	▫	▪	▪	▪	▪	▪
14	15:01-15:30	▪	▪	▪	◦	▪	▪	▪	▪
15	15:31-16:00	▪	▪	▪	◦	▪	▪	▪	▪
16	16:01-16:30	▪	▪	▪	▪	▪	▪	◦	▪
17	16:31-17:00	▪	▪	▪	▪	▪	▪	▪	▪
18	17:01-17:30	▪	▪	▪	▪	◦	▪	▪	▪
19	17:31-18:00	▪	▪	▪	▪	▪	▪	▪	-

Table 3. Shift Type

	Time Slot	Shift
1	8:30-9:00	-
2	9:01-9:30	▪
3	9:31-10:00	▪
4	10:01-10:30	▪
5	10:31-11:00	▪
6	11:01-11:30	▪
7	11:31-12:00	▪
8	12:01-12:30	▪

3. Result

A case study will be conducted to demonstrate how this model will function in the real world. A comprehensive service center run by local governments serves as an example situation.

The data is obtained from the field visit of one local city government service center from the government website for the time period of May 1, 2021, to March 31, 2022. Daily statistics from the network reservation system and the service feedback system were included in the data. The service center has 10 service counters.

On weekdays, all the counters will be in the service state. While on weekends, only counters numbered from 1 to 5 will be serviced. The average service time for one customer is 600 seconds (10 minutes) for all counters on weekdays; thus, $\mu = \frac{30}{10} = 3(people\ per\ 30\ minutes)$. The same situation for mean service time for the service windows on Saturday and Sunday; thus, $\mu = \frac{30}{10} = 3\ (people\ per\ 30\ minutes)$. The average arrival rate Table 4 for a customer is as follows:

Table 4. λ Table.

	Time Slot	λ
1	8:30-9:00	4
2	9:01-9:30	4
3	9:31-10:00	5
4	10:01-10:30	5
5	10:31-11:00	5
6	11:01-11:30	5
7	11:31-12:00	2
8	12:01-12:30	2
9	12:31-13:00	2
10	13:01-13:30	2
11	13:31-14:00	2
12	14:01-14:30	5
13	14:31-15:00	5
14	15:01-15:30	6
15	15:31-16:00	6
16	16:01-16:30	5
17	16:31-17:00	5
18	17:01-17:30	5
19	17:31-18:00	5

Then, this paper solves Models A and B after acquiring the values for the queuing theory parameters and entering them into the model to get the best outcome. The steps for solving Model A are as follows:

a. Optimal Value output

Firstly, we want queuing time to be 5 minutes and set the value for $\rho_t = 0.8\ (t = 1, \ldots, 19)$. Because the service center has fewer workers in the noon (11:30-14:00), assuming the queuing time is 10 minutes, $\rho_t = 0.7\ (t = 8, \ldots, 12)$.

After inputting all the values needed for our model, the result can be calculated:

$$(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8) = (2,2,3,1,0,1,0,1)$$

b. Sensitivity Analysis

Assume the government offers regular training programs for these counter staff, increasing the average service rate for all counters, $\mu = 4$.

After inputting all the values needed for our model, the result can be calculated:

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$$(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8) = (2,1,4,1,0,1,2,5)$$

The same steps for Model B:

a. Optimal Value output

Set a value for $\rho_t \leq 1.0$, because our queue is finite. Thus, this paper chooses $\rho_t = 0.9$.

After inputting all the values needed for our model, the result can be calculated:

$$(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8) = (1,1,2,0,0,1,0,0)$$

b. Sensitivity Analysis

Increase the service rate to $\mu = 4$.

After input all the values needed for our model, the result can be calculated:

$$(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8) = (0,1,1,2,0,0,0,1)$$

4. Discussion

The purpose of this paper is to improve the service rate and quality of service at the service counter and reduce the queuing time of customers in the service hall through the linear programming model of queuing theory. Based on the above modeling and analysis, this paper tried to solve the queuing problem by minimizing staffing, and finally found the number of employees in the best state. Adjusting the number of employees to the most accurate state can optimize the queuing experience of the service. However, this study also has some limitations. For example, it does not subdivide the service industry. Because of their different characteristics, different service industries also have different characteristics in the optimization of queuing problems, which will be improved in future research.

5. Conclusion

This paper is aimed at using the method combining linear programming with queue theory to develop 2 steps to optimize the manpower scheduling problem for a real-world situation. A case study of the local government service counter is provided. After applying our model to the case problem, this paper has achieved one goal: to reduce the number of staff to a more accurate state. Our research finds that there exist some real problems with the service like their service rate is a bit low and needs to improve.

In addition, this model has application value for a wide range of other fields, such as banks, post offices, and other scenarios.

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