Denoising diffusion model as handwritten digit generator

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Abstract. The diffusion model is the process by which variables are introduced and propagated through a population. The equation can be applied to any physical system that exhibits such processes through time. Generally speaking, diffusion occurs when a physical system is connected to another by a small number of interconnected points; the interconnected points' overall connectivity can be represented by a network. The diffusion model is a mathematically defined process used to analyse the movement of particles from a region of high concentration to that of low concentration. Based on the diffusion coefficient and polarization, several studies have established that the equilibrium port-to-port distance can be calculated. The diffusion model is useful for solving the problem of noise in imaging systems, especially when an object has similar properties in all directions. When discussing diffusion, it is essential to refer to the diffusion coefficient. The literatures find denoising diffusion model to involves the process where a pixel value is estimated based on values at surrounding pixels. On the other hand, a forward process is passing through an image and replacing pixels based on their quality estimates. Reconstruction involves reconstructing an image from its components, including the subsamples and low-quality components. This model achieves satisfactory performance on digital number image generation.

Keywords: deep learning, diffusion model, image generation, image reconstruction.

1. Introduction

The diffusion model is a system that generates data similar to the training data and operates by destroying the training data through successive addition of Gaussian noise [1]. Diffusion is a statistical process in which the probability of observation at point p to occur at point q increases with the distance between them. Diffusion models are used to analyse the behaviours of various processes that involve the relationship among variables. Forward models describe data in two dimensions and account for only two diffusion coefficients, which are the simplest diffusion models. In addition to analysing datasets using resampling, researchers have shown that denoising models can also be used to estimate the true signal from noisy data, instead of starting from scratch as with a new dataset every time. The diffusion model in data science describes how new ideas and knowledge spreads across a population [1]. The forward process of the diffusion model describes the initial spread of ideas and expertise as they are passed on from one person to another. The process shows us how the necessary behaviours change can be achieved more quickly as time goes by. The diffusion model describes how knowledge, attitudes, and practices are shared among a group. The diffusion model is a popular application of social network theory [1]. The diffusion process describes how a new idea or innovation moves through a population

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and reaches the final adoption stage. The model also indicates how often a population has observed a disease. The rate is based on how long the infection takes to reach its peak. The denoising diffusion model entails the process whereby a pixel value is estimated based on values at surrounding pixels [2]. The forward process is passing through an image and replacing pixels based on their quality estimates. On the other hand, reconstruction involves reconstructing an image from its components, including the subsamples and low-quality components.

2. Related work

According to the literatures, the diffusion model is a tool used in qualitative research and data analysis that enables the researcher to explore patterns in a particular phenomenon. The diffusion model can be applied to understand better the data within a wide variety of fields, including marketing, economics, and sociology [1]. The diffusion of innovations and the diffusion of scientific knowledge is an ongoing process that varies over time, space, and culture. The concept of innovation originated in sociology and cultural anthropology and gained popularity with its application in other social systems. However, it has also been applied in economics, engineering, and management disciplines to explain how new technology or practices diffuse into a specific region or company. According to Lin et al. [3], the diffusion model also acts as a deterministic-random process, which describes the dynamics of a health epidemic according to the number of infected individuals after exposure.

Harvey et al. [4], identifies that diffusion model is to introduce new and innovative products in the market, and capture customers' attention towards them [4]. The model captures customer attention using marketing strategies such as promotion, advertising, and direct selling. The use of the diffusion model builds on the assumption that political, economic, and social factors form a continuum of importance [4]. The process implies that the essential factor will impact whether or not customers adopt a product. The diffusion model also assumes that consumers move through various stages of the adoption of a product in their decision process. The diffusion model is used to study how people adopt a new product or technology. The model assumes that a new product is introduced into the market and consumers progress through five stages in their decision process: awareness, interest, evaluation, trial, and adoption. Diffusion of Innovation is the process of increasing the use of innovation within a population. Technology diffusion is how new and improved products, processes, and styles spread from one population to another. Diffusion of Innovation and diffusion of technology are related but different concepts [5]. The theory of diffusion explains how, why and at what rate new ideas and technologies spread through society. In addition to looking at the adoption process for individuals, marketers must also consider the effects of group influence on innovation adoption.

3. Method

3.1. Diffusion theory

The diffusion of innovations theory is a framework for studying the adoption and spread of new ideas. It was first proposed by Everett Rogers, who developed the theory in 1962 [6]. The model has been used to study many innovations, from healthcare to education and technology. The diffusion of innovations is a process that occurs in five stages, including awareness, persuasion, decision, implementation, and continuation.

3.1.1. Awareness. The diffusion process is a series of steps people go through when they are exposed to new ideas. The first stage is awareness, where the group introduces the concept. Creating awareness can be done through an external source or within the group. Persuasion is the second stage of the diffusion process, where people are exposed to information about the idea [5]. The exposure can be in the form of advertising, news stories, or word-of-mouth communication. The decision is third stage, where people decide on whether or not to adopt an innovation. The third stage implies buying a product or service, or it could simply be accepting a new way of thinking about something. For an idea to become born by others, it must be spread out and spread further through diffusion. The diffusion process is also

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an opportunity for people to learn about innovation. Either through an external source or from within the group itself.

3.1.2. *Persuasion.* The second step is persuading the audience that the new idea is true. Convincing can be done through persuasion, which is a two-step process, first is finding out what the person already thinks about a given topic, then try to change their mind [5]. Persuading people that an idea is accurate, requires first finding out what they already think about the topic. The results can be achieved by asking them questions or by listening carefully to what they say.

3.1.3. *Decision.* The decision stage is the point at which a person accepts or rejects an innovation. Several factors contribute to decision, including awareness of the new technology, for example, seeing an ad for it. The belief that the technology will provide benefits [6]. Ability to use the technology, Convenience in terms of ease and speed of adoption and use. Ease and speed are two things that may contribute to decisions not being made in the realm.

3.1.4. *Implementation.* The implementation stage is the final stage of the diffusion model, where the idea is put into practice. In this stage, people attempt to make the idea work for them. For an idea or concept to become widely accepted, it must be implemented by many people who use it daily and have success with it, or at least enough [3]. For example, suppose someone has a new invention that makes something easier than before but has not been widely adopted yet.

3.1.5. *Continuation.* The final stage of the diffusion theory is called continuation. The stage can be challenging to achieve because it requires everyone in a group to use it. If trying to get one's idea accepted, it helps if one can get the first person to adopt it. Once that person has adopted it, they will be more likely to tell others about it and try other ways of using it. They may even act as a role model for others because they have already adopted the product or service and are now using it successfully.

3.2. Training

According to Kim et al. [7], diffusion is a continuous time stochastic process whose state space is the real line. It is a case of a continuous-time Markov chain and is learned in many applications, including physics, biology, psychology, and finance [8]. A discrete-time diffusion process is a mathematical model for the spread of information in a population. In the type of diffusion process, individuals are assumed to be independent and identically distributed, with covariance matrices C I. The probability distribution of x I is written as where P (x I) is known as expectation or mean function and its inverse F-1(z) represents autocorrelation function [9]. The quantity p x is called as "probability of observation" that can be calculated using the Bayes rule where p y = probability density function f (x y) [7]. In discrete time models, when only two states are present at each moment, t=0, t = T + to t +N where N represents the interval length between two consecutive observations, then there will only be one outcome variable available, not two as shown in Figure 1. For example, if three competitors compete against each other, only one wins. The probability of observing x I at time t is given where p (x I) represents the probability density function, and F-1(z) is the autocorrelation function.

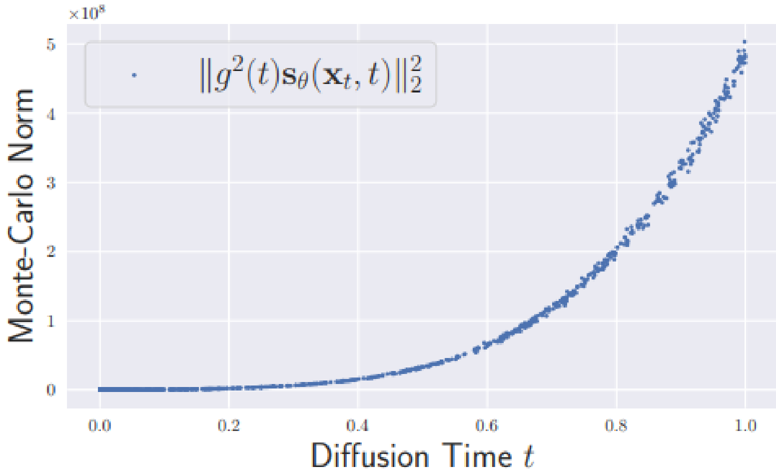

Figure 1. Monte-Carlo normalization at different diffusion time.

3.3. Denoising diffusion model

Denoising diffusion models are associated with dimensionality reduction techniques usually used in machine learning literature. The concepts used in denoising diffusion models borrow from concepts of problem probabilistic methods, which are applicable in most applications [10]. One of the most common methods utilizing the denoising diffusion model is Markov chains. The denoising diffusion modelling involves a two-step forward or reverses the process. The reverse process uses learning the conditional probability densities to eliminate noise. The process involves an essential neural network model which undoes the voice.

Denoising refers to the process of removing noise from an image to improve the quality of the picture. It is often used in images captured with a low-quality camera or compressed to reduce file size [11]. The diffusion model is used for denoising images. The diffusion model performs convolution and then applies Gaussian blur. Convolution takes the input image and applies linear filtering to produce an image that will be blurred with Gaussian blur. The diffusion model is essential in the active learning approach to image denoising. Denoising has a number of applications, such as improving the quality of images captured by low-quality cameras, reducing noise present in noisy images, and filtering out pixel values that are redundant or irrelevant to rendering [11]. Denoising is used on digital photos as well as video frames. In the activity, the images are input into a diffusion model and blurred with a Gaussian filter. The diffusion model converges to the correct solution for an image by iteratively running the Gaussian filters on subsets of input pixels to find areas that contribute the most noise [9]. The denoising diffusion model is a data science algorithm that removes noise from a data set. Noise is irrelevant information that can distort the data set and make it challenging to analyse. The denoising diffusion model takes in a noisy input signal and tries to find the underlying signal by first passing the input signal through an inverse filter, which filters out noise but preserves the original information. Secondly, the filtered signal is then passed through an anti-noise filter, removing any residual noise, and eliminating some of the original information. The final output of the process is a clean signal with reduced noise levels [8]. Finally, the inverse filter acts to remove noise, while the anti-noise filter eliminates any residual noise. The combined output is a clean signal with reduced noise levels

3.4. Training

The process of data science is complicated. It is not just about collecting data, but also about analysing it, and finally, the use of the data to make decisions and predictions. Data scientists are concerned with business intelligence and information transparency, which is vital to their job [9]. The process is accomplished using statistical methods to analyse data sets and build models predicting outcomes. The

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forward process entails data collection, data analysis, and decision-making. Data collection involves collecting all relevant information from all possible sources and ensuring that there is no missing information. The data analysis stage includes many tasks like exploration, summarization, visualization, and modelling. The decision-making stage includes tasks like interpreting results, evaluating trade-offs and risks, and determining actions based on the results or findings from previous steps. The forward process operates like a Markov chain instead of using a recorder in the VAEs, which do not require training. Using the forward process, the time depends on the immediate predecessor, which is t – 1 [9]. The probability density is that affected by the initial time.

3.5. Reconstruction

Reconstruction in data science is converting data stored in one format to another [12]. Data can be reconstructed in many ways. One way is by using a mathematical operation. Another way is by using an algorithm that converts the data into different formats. An algorithm is a set of rules or instructions for a task, especially in mathematics and computing. In data science, an algorithm is a process that transforms input data into the desired output. An algorithm can be implemented using machine learning techniques. Algorithms are usually classified as either deterministic or probabilistic [13]. A deterministic algorithm is a process that always takes the same output given the same input. The iterative function to calculate the factorial of a number, n!, is an example of a deterministic algorithm.

4. Result

The reconstruction in the diffusion model assumes that a person is only interested in finding a product he considers fair quality with a realistic price [14]. Estimating the value of brand equity for each brand is possible by performing calculations similar to standard market valuation methods using assumptions [8]. However, to use diffusion model effectively, the following assumptions must be met: (1) the sample comes from a target market with equal brand awareness, experience, and preference for all products. (2) the individual possesses information about all products available at a comparable price level. (3) all other characteristics remain constant. In the reconstruction process the estimating of the probability density is followed by the parametrization of the mean function. The mean function is computed using the formula:

$$\mu_\theta(x_t, t) = \frac{1}{\sqrt{a_t}}\left(x_t - \frac{\beta_t}{\sqrt{1-\bar{a}_t}}\ \epsilon_\theta(x_t, t)\right) \tag{1}$$

The formula $\mu_\theta(x_t, t)$, x represents the space variable, t represents time, α represents the variance, β represents hyper parameter and θ represents learned weight.

According to Harvey et al. [4], the reconstruction step is a significant and essential part of the diffusion model as shown in Figure 2. Knowing how to reconstruct the diffusion curve helps understand how the model is correctly set up and gives insight into why data values are where they are. An excellent way to reconstruct the model is by beginning with a 3-step process. First, determine which process is located at each node [10]. Second, decompose the process into its constituent parts by creating more than one outlet from that node. Lastly, plot out the outlets from most to least influential in identifying which leads to new products because it has more elements than any other.

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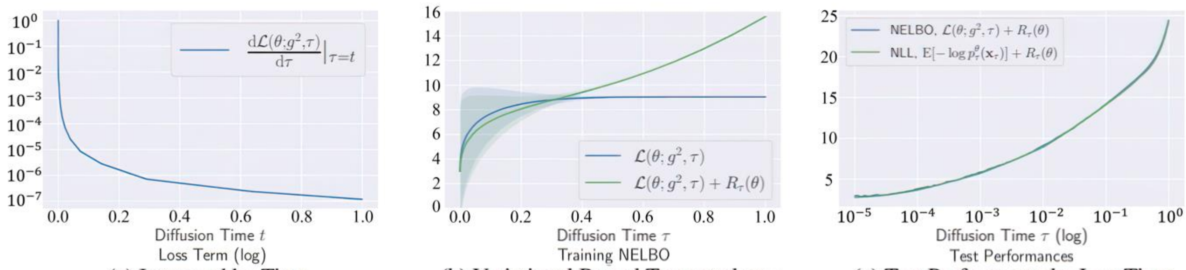

(a) Integrand by Time (b) Variational Bound Truncated at τ (c) Test Performance by Log-Time

Figure 2. Training process and the performance.

The diffusion model output for digital number generation in different epochs is demonstrate in Figure 3.

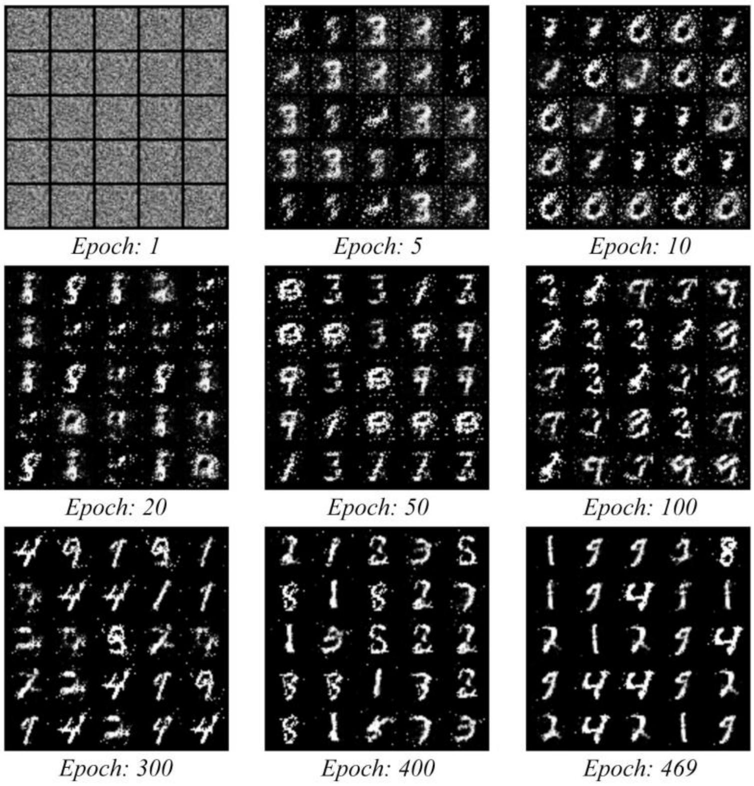

Figure 3. Result of the generated digital numbers.

5. Discussion

In this paragraph the author sequentially discusses the advantages and disadvantages of the diffusion model.

The diffusion model of innovation is an essential concept in the management field. The diffusion of creation is introducing new ideas and improved methods into a society. It indicates that a product or service spreads from a source, which is most often another company, to other users through word-of-mouth or other oral communication, as well as by demonstration and experimentation [15]. The diffusion of innovation is an essential concept in the management field. The model indicates that a product or service spreads from a source, which is most often another company, to other users through word-of-mouth or other oral communication, as well as by demonstration and experimentation. The theory holds that new ideas take time to catch on and spread. For example, when a new product is introduced, people may need time to learn about it and decide whether they want to purchase it. The theory holds that new ideas take time to catch on and spread. According to Harvey et al. [4], the diffusion model is the simplest of all models. Since it is a mechanical model, it ignores the price elasticity of demand. The model also assumes that there are no barriers to entry. The advantages of the model are that it can also be used to explain viral disease spread as a result of social influences as well as physical contact.

Although the diffusion model can be used to study the impact of one product on another, the model cannot be used to explain the movement of ideas or products in society. The diffusion theory explains how new thoughts, behaviors, or consequences spread through a population. It is based on the idea that people adopt new beliefs or behaviors when they see others doing it. It assumes that the diffusion of information is driven by certain factors, such as time and social networks [13]. The diffusion model is often used to explain how information spreads, but it fails to explain why people adopt new ideas, behaviors, or products.

The model does not consider the role of hosts, such as livestock, who may deposit viruses into the environment [16]. Additionally, the assumption is that all people will be exposed to a given pathogen with equal probability. The diffusion model also does not consider the role of social networks, which may be more or less important in a given situation. The diffusion model has been widely used to explain the spread of diseases such as influenza and cholera, but it can also be applied to many other types of information [5]. The diffusion model only describes how ideas or products become successful over time. The model also fails to account for how, when, and where the ideas are introduced.

6. Conclusion

The diffusion model is a social science model that describes how new products or innovations spread throughout a given population. Everett Rogers developed the model and later was updated and expanded upon by many other social scientists. The four stages of innovation are the introduction, adoption, maintenance, and decline phases. The model applies to any product or idea, like technology, when introduced into a specific culture. The four-stage diffusion innovation model is a popular way to explain how new ideas spread through a population. It was first published by Everett Rogers in 1962 and updated by Joseph R. Schumpeter in 1942, who further developed the concept. The model is based on the idea that people do not change their behaviours until they get convinced that the innovation is worth it. If people are not interested in adopting innovation, it will not spread easily throughout a population. However, suppose there is enough demand for an innovator's product or service. In that case, this could lead to quick adoption of their product or service once it becomes available on market shelves.

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