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The Relationship between Air Resistance and the Vibrating Period of Simple Pendulum

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Abstract. The simple pendulum is a typical model of the compound pendulum. In this model, resistance and buoyancy are negligible. This paper researches how air resistance affects the vibration period in the simple pendulum model but with exact conditions. There are two simplified air resistance models that can apply in the models, one is the first power air resistance model, and the second is the second power air resistance model. But the first power air resistance model is quite complex, there is no equipment to measure its coefficient of it, so it only applies the second power air resistance. This paper studies the formula that revealed air resistance and related suitable cases. The results show that the time from the formula is longer than the experiment a little bit.

Keywords: Compound pendulum, Air resistance, Vibration period, Resistance coefficient.

1. Introduction

A simple pendulum is like an idle model used as a reference to another pendulum which does use for reality. A compound pendulum has a finite weight and shape and is used as a natural pendulum. As the experiment can't neglect the air resistance, and needs an accurate vibrating period, although this paper researches the simple pendulum model, it is a compound pendulum model [1]. This paper will explore the simplified models of air resistance and use the simplified models to speculate the different vibration periods of a compound pendulum with consideration of the air resistance formula in simplified models. At low speeds, which occur through viscous materials, the fluid layers drag surrounding layers along and transfer momentum to each subsequent layer at a rate proportional to its velocity. At high speeds, the momentum that is imparted to each fluid particle is proportional to the speed of the object. The number of particles of the fluid encountered every second is also proportional to the speed [2]. But, the coefficient in the first power model has no equipment in the laboratory to measure it, so this paper only researches one model. The air resistance model assumes that the resistance is proportional to the square of the velocity. It is easy from the experiment result, no air resistance model, and air resistance model in a figure to compare the results. This paper speculates the resistance model. Comparing the model with the experimental results in a figure to directly analyze the experiment, and give a conclusion. This paper can direct explain how air resistance affects the vibrating period of the pendulum. People can direct know the air resistance in the formula, to calculate the time of a period. As there is a much bigger random error for experimenters who only count one time, so they need to count ten periods, but it has a little bit different, as the period may change with the number of periods. This paper's conclusion can directly give the time of a period, and compare the difference between exact and formula. Apart from that, this paper only researches one air resistance model, which is the second power model, so, the reader can know whether the second power model is appropriate for this case in this paper.

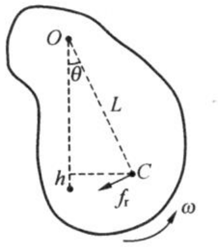

Figure 1. Schematic diagram of compound pendulum vibration [3].

2. Speculation of second power air resistance model in time of pendulum

Let pendulum mass be M and oscillate in the vertical plane about the horizontal axis passing through a point O. The distance from axis O to the centre of mass C is L, and the Angle between OC and the plumb line is θ when the complex pendulum swings at angular velocity ω. Because of the air resistance, the motion of the complex pendulum is no longer an ideal dissipative motion. Assume that the air resistance to a complex pendulum is proportional to the square of the velocity of motion, then:

$$f_r = -kv^2 = -kL^2\omega^2 \tag{1}$$

k is the coefficient air resistance at this model. Air resistance does negative work on the compound pendulum, according to the principle of function can get:

$$\frac{1}{2}I\omega^2 + Mgh - \int f_r L d\theta = 0 \tag{2}$$

Plug formula (1) to (2), derive it with t, and get:

$$\frac{d^2\theta}{dt^2} + \frac{kL^3}{I}\left(\frac{d\theta}{dt}\right)^2 + \frac{MgL}{I}\theta = 0 \tag{3}$$

Making $\beta = \left(\frac{d\theta}{dt}\right)^2$,then:

$$\frac{d\beta}{d\theta} + \frac{2kL^3}{I}\beta + \frac{2MgL}{I}\theta = 0 \tag{4}$$

Solute the last formula can get:

$$\beta = c_0 e^{-\frac{2kL^3}{I}\theta} + \frac{MgL}{2k^2} - \frac{Mg\theta}{aL^2} \tag{5}$$

c_0 is the constant of integration. When $\theta = \theta_0$(the maximum amplitude), angular speed is $\left(\frac{d\theta}{dt}\right)_{\theta=\theta_0} = 0$, plug it to last formula can get:

$$c_0 = \frac{\frac{MgL}{2k^2L^5}\left(\frac{2kL^3\theta_0}{I} - 1\right)}{e^{-\frac{2kL^3}{I}\theta_0}} \tag{6}$$

And a given that $\beta = \left(\frac{d\theta}{dt}\right)^2$, it can get:

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$$t = t_0 + \int \frac{d\theta}{\sqrt{\beta}} = t_0 + \int \left(c_0 e^{-\frac{2kL^3}{I}\theta} + \frac{MgL}{2k^2L^5} - \frac{Mg\theta}{kL^2} \right)^{-\frac{1}{2}} d\theta \tag{7}$$

t_0 is the constant of integration. As θ_0 and k both are small, so use Taylor's theorem due with $e^{-\frac{2kL^3}{I}\theta}$

$$e^{-\frac{2kL^3}{I}\theta} \approx 1 - \frac{2kL^3}{I}\theta + \frac{2k^2L^6}{I^2}\theta^2 \tag{8}$$

Plug it into formula (7) can get:

$$t - t_0 = -\frac{I}{2kL^3}\sqrt{-\frac{2}{c_0}}\, sin^{-1}\left[\frac{\frac{2c_0kL^3}{I}\theta - \left(c_0 + \frac{MgL}{2k^2L^5}\right)}{\sqrt{\left(\frac{MgI}{2k^2L^5}\right)^2 - c_0^2}} \right] [3] \tag{9}$$

Making only θ on the left side

$$\theta = \frac{c_0 I + \frac{MgI^2}{2k^2L^5}}{2c_0kL^3} - \frac{I\sqrt{\left(\frac{MgI}{2k^2L^5}\right)^2 - c_0^2}}{2c_0kL^3} \times sin\left[\frac{2kL^3}{I}\sqrt{\frac{-c_0}{2}}(t - t_{0)} \right] [3] \tag{10}$$

The angular frequency on this model is that:

$$\omega = \frac{2kL^3}{I}\sqrt{-\frac{c_0}{2}} \tag{11}$$

Plug c_0 to last formula can get:

$$\omega = \sqrt{\frac{\frac{MgL}{I}\left(1 - \frac{2kL^3\theta_0}{I}\right)}{1 - \frac{2kL^3}{I}\theta_0 + \frac{2k^2L^6}{I^2}\theta_0}} \tag{12}$$

Therefore, the vibrating period of compound pendulum at this model is that:

$$T_1 = 2\pi\sqrt{\frac{1 - \frac{2kL^3}{I}\theta_0 + \frac{2k^2L^6}{I^2}\theta_0}{\frac{MgL}{I}\left(1 - \frac{2kL^3\theta_0}{I}\right)}} = T_0\sqrt{1 + \frac{2k^2L^6\theta_0}{I^2 - 2kL^3I\theta_0}} \tag{13}$$

Obviously, the vibrating period is longer when it considers about the air resistance.

The second power model reflects that air resistance is longer the vibrating period. As the moment of inertia is that $I = M(R + L)^2$, R is the radius of bob, plug it to formula (13) get that:

$$T_1 = 2\pi\sqrt{\frac{1 - \frac{2kL^3\theta_0}{M(R+L)^2} + \frac{2k^2L^6\theta_0}{M^2(R+L)^4}}{\frac{gL}{(L+R)^2}\left(1 - \frac{2kL^3\theta_0}{M(L+R)^2}\right)}} \tag{14}$$

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As see from formula (14) that, T_1 dependent on the Distance from center of mass to axis of rotation (L) and the coefficient of drag (k). when the coefficient of damping equal 0, $T_1 = T_0$.

3. Methodology

3.1. The experimental design and experimental procedure

The experiment in this paper used a simple pendulum model to count ten periods with different lengths of spring ten times, as counting one period has a much bigger random error. This experiment is quite simple, it only used a timer, a simple pendulum model, and a ruler. Before doing the experiment, it needs to measure the length of the spring in a more accurate value the trial needs. After that pick up the bob to the 5° amplitude, 5° amplitude is a constant, it cannot change. Then, removing the hand, making the bob do a free fall motion, meanwhile, click the timer to count. In addition, the bob does the last period, prepares to stop the timer. The bob finished last period, meanwhile, click the timer, and recording the result.

3.2. The results of the experiment

Table 1. The result of the experiment.

<table>
<tr><th>The time of ten periods ±0.01/s
The length of bob ±0.05/cm</th><th>Trial 1</th><th>Trial 2</th><th>Trial 3</th><th>Trial 4</th><th colspan="3">Trial 5</th><th colspan="2">Average time of one period</th></tr>
<tr><td>10</td><td>6.68</td><td>6.69</td><td>6.66</td><td>6.68</td><td colspan="3">6.70</td><td colspan="2">0.67</td></tr>
<tr><td>15</td><td>8.10</td><td>8.08</td><td>8.12</td><td>8.10</td><td colspan="3">8.12</td><td colspan="2">0.81</td></tr>
<tr><td>20</td><td>9.23</td><td>9.26</td><td>9.26</td><td>9.27</td><td colspan="3">9.29</td><td colspan="2">0.93</td></tr>
<tr><td>25</td><td>10.26</td><td>10.28</td><td>10.25</td><td>10.29</td><td colspan="3">10.24</td><td colspan="2">1.03</td></tr>
<tr><td colspan="4">30</td><td>11.27</td><td>11.29</td><td>11.31</td><td>11.23</td><td>11.25</td><td>1.13</td></tr>
</table>

It needs the coefficient of drag force (k),$F_r = \frac{1}{2} C_d \rho A v^2$, the k is the $\frac{1}{2} C_d \rho A$, C_d is the drag coefficient, which is 0.5 for sphere [4-5], ρ is the density of the medium, A is the cross-sectional area. After the calculation, the coefficient of drag force is 1.86×10^{-4} kg/m. Now, it is given that the mass of bob is 0.032 kg, R is 0.19m, g is 9.80 m/s2, θ_0 is 5° or 0.087 radian, k is 1.86×10^{-4} kg/m. Then, comparing three models.

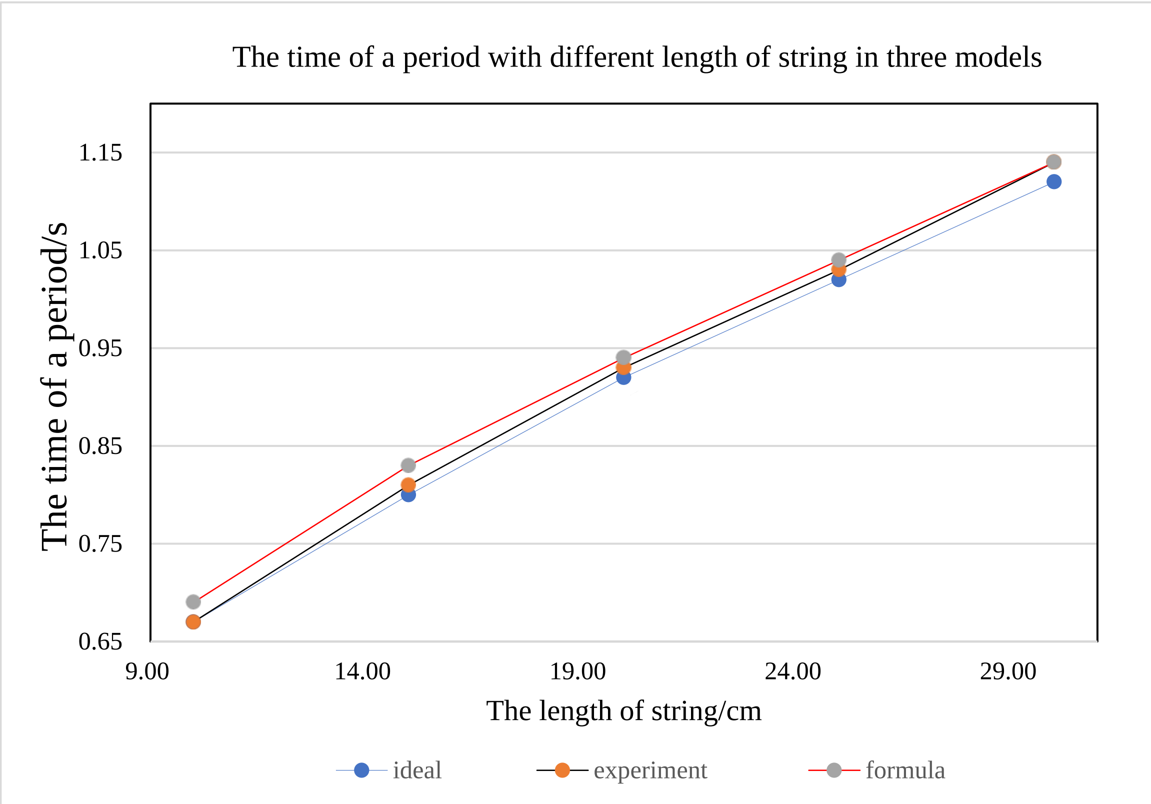

Figure 2. The comparation of three models.

4. Discussion of the experiment

In this paper, is going to research the air resistance in the formula, and which case it is more suitable. From the figure 1, the experiment value and the ideal value more much closer between $9 < L < 25$cm, and the same between 10 to 11 cm. It shows that, small amplitude at smaller length of string receive low drag force, it can neglect the drag force in this case. In addition, that is why formula value are far away with experiment value, but it more suit for longer length of string, for length bigger than 25cm, the experiment value is begin to close the formula value. To a conclusion, that the formula derived from this paper is more suitable to apply for length bigger than 25cm, as the bob received a low air resistance force at small amplitude and smaller length. The length of string between 10 to 11cm can apply the ideal formula, it is more suitable at that case, as the reason already talk about. Furthermore, the length from the centre mass to the pivot is length of string plus the radius if sphere. It is pulsed at the process of experiment, but it is not revealed at the analysis.

5. Conclusion

This paper explored the relationship between the length of string and the time of period under the conditions have air resistance. It is can be seen from the paper, the formula of the topic is derived. After that, this research can be advanced by adding the first power model. The first power model is a low speed case, it is suited for this case. But, there is second power model in this case. After exploring the first power model, comparing the results from the ideal formula ($T = 2\pi\sqrt{\frac{L}{g}}$), experimental results, first power model results and second power model results to give a further analysis.

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