Disney visitor problem: Integer optimization using enumeration method and set covering problem

Ludwig Lin[1,3,4], Kajiwara Aika[2]

[1]Shanghai Yangpu Bilingual School, Shanghai, China, 553621949@qq.com
[2]Suzhou Foreign Language School, Suzhou, China, 3147804575@qq.com
[3]Correspondence author

[4]553621949@qq.com

Abstract. For Disneyland visitors, a well-designed route is often necessary to experience the maximum number of preferred entertainment facilities within a limited time. To construct the best way that optimizes visitors' satisfaction, a survey is first conducted to estimate the attraction value of each facility, followed by the collection of data that record the traveling time among each facility and the waiting line time. Using collected data and listed constraints, a possible route is listed as an example. To solve the problem, a model is constructed based on integer linear programming. The original, incomplete, and modified formulations are listed in the last part of this paper.

Keywords: Integer Linear Programming, Optimization, Disney Visitor Problem.

1. Introduction

With China's population rise, many people are jammed in most recreational places. People cannot have a good experience due to traffic jams, long queues, and crowding. Under the circumstances, preparing and designing a detailed plan that records the best route is necessary. For example, we choose a typical metropolitan city-shanghai and Disneyland, to be our fun places.

When people visit Disneyland in Shanghai, they may be annoyed since they must line up for the recreation facilities, which limit the items available each time, reducing satisfaction. Therefore, effectively arranging the waiting time and walking routes is essential to maximize visitors' satisfaction. On the other hand, this research paper focuses on the waiting line problems and the dominant time cost in Shanghai Disneyland to offer visitors an enjoyable experience.

In this paper, 13 recreation facilities were under consideration ——TRON light cycle Power Run, Soaring over the Horizon, Rex's Racer, Peter Pan's flight, Seven Dwarves Mine Train, Alice in Wonderland Maze, Once Upon a Time" Adventure, Pirates of the Caribbean battle for the sunken treasure, Fantasia Carousel, Jet packs, Roaring rapids, Slinky Dog Spin, and Camp Discovery. An assumption of spending 5 minutes on each item is made. Meanwhile, this paper only considers weekdays as the day of traveling rather than the weekend, requiring longer queues. To research the attraction value of each item, a survey is designed and conducted thereby, and the average value of each attraction value is taken. Moreover, several tables are created, including the average walking time between each facility, waiting time, and attraction value of each facility. We aim to seek appropriate methods to solve the

"Disneyland problem" that optimizes the attraction value within a limited time —— 10.5 hours. More constraints are listed in the next part of this essay.

2. Background and Problem Set-up

We randomly provided surveys to Chinese people who had previously traveled to the target Disneyland and computed the mean attraction value corresponding to each facility. We also visited the official website, which provides average waiting time and walking time among facilities to its tourists; in this case, we will just assume the experiencing time for each facility to be 5 min. People will find that the walking time is longer than expected because plenty of people are in Disneyland, which contributes to slowing our pace. Applying these statistics we gathered, we will substitute them into a linear programming formulation to work out the most satisfying solution to enjoy the attraction for prospective tourists. In conclusion, the three essential variables are:

1) the traveling time between facilities
2) the experience time of each facility (with an assumption of 5 min)
3) the attraction value(satisfaction) of each facility

Table 1. Walking time between each facility.

	gate	1	2	3	4	5	6	7	8	9	10	11	12	13
gate	— —	27min 1.4km	17min 950m	4min 220m	15min 860m	10min 580m	11min 603m	13min 725m	5min 281m	23min 1.3km	26min 1.4km	14min 755m	18min 1km	15min 842m
1	— —	——	18min 1km	10min 530m	9min 490m	12min 680m	12min 661m	10min 560m	15min 835m	9min 523m	3min 192m	14min 753m	8min 422m	15min 840m
2	— —	——	——	27min 1.4km	15min 840m	11min 590m	11min 612m	12min 676m	5min 322m	9min 495m	16min 866m	4min 234m	18min 1km	2min 151m
3	— —	——	——	——	7min 350m	11min 640m	16min 852m	19min 947m	15min 808m	38min 2.1km	9min 478m	23min 1.2km	2min 211m	24min 1.3km
4	— —	——	——	——	——	5min 280	4min 258m	3min 163m	10min 562m	8min 444m	8min 435m	11min 600m	2min 211m	12min 687m
5	— —	——	——	——	——	——	1min 19m	2min 141m	5min 284m	6min 366m	10min 545m	7min 401m	7min 410m	9min 488m
6	— —	——	——	——	——	——	——	2min 121m	5min 303m	6min 376m	9min 525m	7min 420m	8min 449m	9min 507m
7	— —	——	——	——	——	——	——	——	7min 396m	5min 319m	7min 413m	7min 427m	6min 375m	9min 514m
8	— —	——	——	——	——	——	——	——	——	6min 367m	13min 699m	6min 335m	13min 714m	5min 307m
9	— —	——	——	——	——	——	——	——	——	——	7min 387m	4min 246m	10min 566m	6min 333m
10	— —	——	——	——	——	——	——	——	——	——	——	11min 617m	6min 371m	13min 704m
11	— —	——	——	——	——	——	——	——	——	——	——	——	15min 831m	1min 72m
12	— —	——	——	——	——	——	——	——	——	——	——	——	——	17min 918m
13	— —	——	——	——	——	——	——	——	——	——	——	——	——	——

1. TRON light cycle Power Run
2. Soaring over the horizon
3. Rex's Racer
4. Peter Pan's flight

5. Seven Dwarves Mine Train
6. Alice in Wonderland Maze
7. Once Upon a Time" Adventure
8. Pirates of the Caribbean battle for the sunken treasure
9. Fantasia Carousel
10. Jet packs
11. Roaring rapids
12. Slinky Dog Spin
13. Camp Discovery

Table 1 shows the walking time between each facility and the gate. The first row and column are the names of the recreational facilities, and for every two facilities, we only count the walking time once, so you find that some are white space. Noticing that the longest walking time is 38 minutes from Fantasia Carousel to Rex's Racer, walking time becomes an essential factor influencing the visitor's satisfaction. On the other hand, some walking times are less than 10 minutes, so choosing the next facility properly can save visitors much time.

Table 2. Waiting time and Attraction value.

Attraction No.	Attraction Name	Waiting time(min)	Attraction value (/10)	Experiencing time(min)
1	TRON light cycle Power Run	47	10	5
2	Soaring over the horizon	80	7	5
3	Rex's Racer	50	7	5
4	Peter Pan's flight	28	6	5
5	Seven Dwarves Mine Train	6	7	5
6	Alice in Wonderland Maze	5	6	5
7	Once Upon a Time"Adventure	7	5	5
8	Pirates of the Caribbean battle for the sunken treasure	15	7	5
9	Fantasia Carousel	18	5	5
10	Jet packs	50	7	5
11	1Roaring rapids	70	9	5
12	Slinky Dog Spin	35	7	5
13	Camp Discovery	45	5	5

Table 2 demonstrates thirteen recreational facilities' waiting times, attraction value, and experience time. Discovering that some facilities like Soaring Over the Horizon and Seven Dwarves Mine Train cost much time to line up, we balance the waiting time and attraction value, observing that Alice in Wonderland Maze has the most attractive value per minute. However, its attraction value is only 6 out of 10.

One of our group members proposed that if one visitor always enjoys the Alice in Wonderland Maze, he could maximize his satisfaction. However, we opposed this circumstance of decreasing marginal utility. Eventually, we decided that each recreational facility can be only experienced once at most. In this way, people can always get the most satisfaction by sharing the facility for the first time.

3. Constraints

3.1. Time Constraints

1. Total time in Disneyland is ten hours and a half. People can enter Disneyland at 9.00 am and watch the fireworks at 8.30 pm. Apart from mealtimes, it is plausible for tourists to spend 10.5 hours enjoying different recreational facilities.

2. The time for experiencing one recreational facility is 5 minutes.

3. The waiting-line time is different for each recreational facility, and each visitor is supposed to start from the entrance of Disneyland. Specific data are in the above chart.

4. The walking time between every two recreational facilities is also considered. Specific data are in the above chart.

3.2. Quantitative Constraints

5. Tourists are supposed to experience more than or equal to seven recreational facilities.

6. Each recreational facility can only be experienced at most once to prevent decreasing marginal utility.

4. Methodology

A possible method is to construct all routes. To be more specific, list all the possible ways by considering the constraints. As shown in Figure 1, Figure 2 and Table 2.

We tried a route with only five locations, and then we wrote down one of the possible routes in Figure 1.
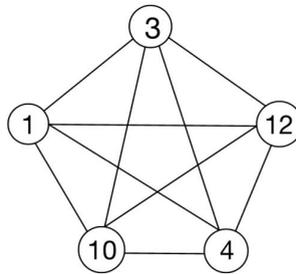
Figure 1. A simple route connection.

In Figure 1, recreational facilities are the graph's vertices, the path is the graph's edges, and a path's walking time is the edge's weight. It is a minimization problem starting and finishing at a specified vertex after having visited each other vertex exactly once, which is a traveling salesperson problem [1]

Also, there are some applications of the traveling salesperson problem. The traveling salesperson problem arises in many different contexts. This paper reports on typical applications in computer wiring, vehicle routing, clustering, and job-shop scheduling. Formulating a traveling salesperson problem is the simplest way to solve these problems. Most applications originated from real-world problems and thus seemed particularly interesting. Illustrated examples are provided with each application. **[2]**
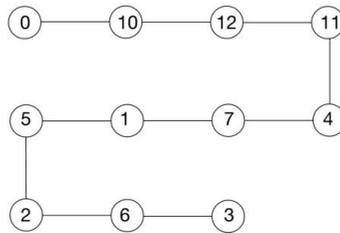
Figure 2. A Plausible Route.

Table 3. Waiting time and experiencing time.

Facility No.	Waiting time	Experiencing time
Gate	0	0
10	50	5

Table 3. (continued).

12	35	5
11	70	5
4	28	5
7	7	5
1	47	5
5	68	5
2	80	5
6	5	5
3	50	5
	440	50

In Figure 2, we choose the recreational facilities: TRON light cycle Power Run, Soaring over the Horizon, Rex's Racer, Peter Pan's flight, Seven Dwarves Mine Train, Alice in Wonderland Maze, Once Upon a Time" Adventure, Jet packs, Roaring rapids, and Slinky Dog Spin. Total time is 440minutes (waiting time) + 50minutes (experiencing time) + 121minutes (walking time) =611 minutes < 630 minutes (constraint); total satisfaction value is 71; number of recreational facility is 10 > 7 (constraint).

Limited by the constraints of total time and only one experiencing opportunity, many routes are invalid, and if we keep on listing plausible ways, it is possible to find the optimal solution.

5. Previous Formulation

5.1. Decision variables
Before we formulate the DVP problem into equations, the definition of mathematical notations and decision variables are needed:

- m = the number of recreational facility ($7 \leq m \leq 13$)

- n = the number of routes

- i = recreational facility number, $i = \{x | 1 \leq x \leq 13, x \in z\}$

- j = route number, $j = \{x | 1 \leq x \leq n, x \in z\}$

- T_j=spending time of route number j

- W aiting time of recreational facility i

- S_i = the satisfaction of recreational facility i

- T_I = walking time between i_a to i_b

- X_j = 1 if the route is selected; 0 otherwise.

- $a_{i,j}$ = 1 if recreational facility i is part of route j; 0 otherwise.

5.2. Set Problem
All the attraction values are recorded in Table 2, and we must add all the attraction values of the facility that the visitor experiences. For the optimal equation, we have to maximize the total satisfaction value.

Maximize {1} $\sum_{i=1}^{m} a_{ij} \ S_i$ i=$\{x | 1 \leq x \leq m, x \in z\}$

Subject to {2} $\sum_{i=1}^{n} T_j X_j \leq 630$

{3} $\sum_{i=1}^{m} a_{ij} W_i + \sum_{i=1}^{m} a_{ij} T_i + 5m \leq 630$, m=$\{x | 7 \leq x \leq 13, x \in z\}$

In the next step, we expand the equation-add up all the attraction value

$$\textbf{Maximize\{4\}} \; Z =$$
$$10a_i1_j+7a_i2_j+7a_i3_j+6a_i4_j+7a_i5_j+6a_i6_j+5a_i7_j+7a_i8_j+5a_i9_j+7a_i10_j+9a_i11_j+7a_i12_j+5a_i13_j$$
$$\textbf{Subject to \{5\}} \; \sum_{i=1}^{n} T_j X_j \leq 630$$
$$\{6\} \; \sum_{i=1}^{m} a_{ij} W_i + \sum_{i=1}^{m} a_{ij} T_i + 5m \leq 630, \; m=\{x \,|\, 7 \leq x \leq 13, x \in z\}$$

However, there's something wrong with the variable, which is redundant regarding the si. It is unnecessary to count all facility's satisfaction and sum them up just to calculate the route j's attraction. Our professor Ming Gu made a correct and more specific equation.

5.3. Corrected Formulation

We formulate the **Disney Visitor Problem** (DVP) as an integer linear program (ILP). Let V_0 be the entrance and V_{n+1} be the exit, with n being the number of attractions, and denote V_1 , $\cdots$, V_n as the attractions.

We consider the following setup:
- $T_{i,j}$ is the time it takes to go from V_i to V_j
- $x_{i,j}$ is the decision variable indicating whether the visitor goes from V_i to V_j
- T_j is the time it takes to play at V_j
- S_j is the satisfaction of playing at V_j

Thus,
- $\sum_{i=0}^{n} x_{i,j}$ indicates whether the visitor goes to V_j , with the total satisfaction $\sum_{j=1}^{n} S_j \sum_{i=0}^{n} x_{i,j}$
- $\sum_{j=1}^{n} x_{0,j}=1$ indicates that the visitor goes to some attraction from the entrance.
- $\sum_{i=1}^{n} x_{i,n+1}=1$ indicates that the visitor leaves the exit.
- $\sum_{i=0}^{n} x_{i,v}=\sum_{j=0}^{n} x_{v,j}$ indicates that the visitor leaves an attraction after the visit.

Putting together, we have the following ILP:

$$max \; \{7\} \; \sum_{j=1}^{n} S_j \sum_{i=0}^{n} x_{i,j}$$

$$s.t. \; \{8\} \; \sum_{j=1}^{n} x_{0,j} = 1$$

$$\{9\} \; \sum_{i=1}^{n} x_{i,n+1} = 1$$

$$\{10\} \; \sum_{i=0}^{n} x_{i,v} = \sum_{j=1}^{n+1} x_{v,j}, \quad v = 1, \dots, n$$

$$\{11\} \; \sum_{i,j=0}^{n+1} T_{i,j} x_{i,j} + \sum_{j=1}^{n} T_j \sum_{i=0}^{n} x_{i,j} \leq T$$

$$x_{i,j} \geq 0, \quad i,j = 0, \dots, n+1$$

We set $xi,i = 0$ above for all i so the visitor does not stay at any attraction after the visit.

With the first three equations, it is known that the solution will always be an integer, but for the fourth equation, we cannot make sure whether its solution is an integer. This is an integer linear program, so the solution should be integers. There may be some constraints under which our solution will always be an integer. We will use minute instead of hour as our unit to avoid situations where the solution is not an integer.

6. Improvements

● The equation that we finally corrected is precisely the [knapsack problem, which means given s set of items, each with a weight and a value, determine the number of each item included in a collection so that the total weight is less than or equal to a given limit and the total value is as significant as possible.]

[3]. Here are some applications of the Knapsack problem. [In its most general form, the nonlinear knapsack problem will be defined as a nonlinear optimization problem with just one constraint, bounds on the variables, and, in some cases, a set of specially structured constraints such as generalized upper bounds (GUBs). This problem is encountered either directly or as a subproblem in a variety of applications, including production planning, financial modeling, stratified sampling, and capacity planning in manufacturing, health care, and computer networks] [4]. However, the circumstance has the limitation of time, so it is hard to solve. We must still consider whether the time congestion can be considered a time knapsack.

● The alternative method is to solve it with dynamic programming. Several items and times should be considered. It is an intractable problem, but now it is solvable.

● I want to work further on this interesting Disneyland problem(maximize enjoyment problem), [The EM (expectation-maximization) algorithm is ideally suited to problems of this sort in that it produces maximum-likelihood (ML) estimates of parameters when there is a many-to-one mapping from an underlying distribution to the distribution governing the observation. The EM algorithm is presented at a level suitable for signal-processing practitioners who have had some exposure to estimation theory.] [5] and I will talk with the Academy and learn how to take a further step with it

7. Conclusion

Our purpose is to make people maximize their satisfaction in a limited amount of time, so we focus on queuing and walking time to let them experience as many recreational facilities as possible.

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