

The Development of the Galactic Rotation Curve

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Abstract. After a period of scientific research practice, the author has acquired some relevant knowledge, and it is hoped to have the Milky Way rotation curve of the gradual improvement process presented. The author uses the literature method to summarize the history of the study of the galactic curve, and show the process in the relative descriptive discourse.

Keywords: Galactic rotation curves, Kepler curve, index plate models, dark matter halos, MOND.

1. Introduction

The galactic rotation curve refers to a functional image of the distance from the galactic center and the velocity of an object in the galaxy [1]. As the Earth rotates, through the radio telescope, the powers corresponding to different frequencies of rays on different channels of silver can be obtained. By Doppler effect [2], the relationship between the speed and power of matter relative to the Earth's motion is established. Then, based on basic geometry and the Sun's velocity relative to the local stationary standard, the actual data link between the distance from the Galactic Center and the speed at which the object is moving is obtained. However, the link between theory and practice is not smooth. The initial images, calculated using simple Kepler theory [3], were far from the actual data. Therefore, people began to consider the establishment of a new theory to solve this contradiction.

The author consulted the relevant literatures, tried to use the description of the review to clarify the theory of the galaxy rotation curve more clearly and simply, found the loopholes and made further improvement, and finally made it consistent with the reality.

2. The Main Body

2.1. Observation

To measure the rotation curve of the Milky Way. The author generally scans the radio band drift on the ground, and records the power of the radio band at all positions of the meridian as the Earth rotates.

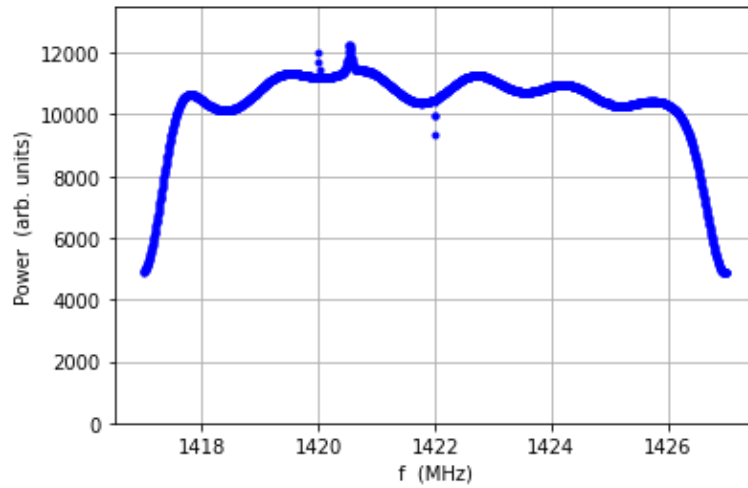


Figure 1. Power diagram at different frequencies (credit to Professor Daniel Marlow).

By analyzing the narrow but pronounced 21-centimeter lines of hydrogen in the band, based on their original frequencies and Doppler effect, it is found that the relative velocities of the objects that emit the rays and the earth.

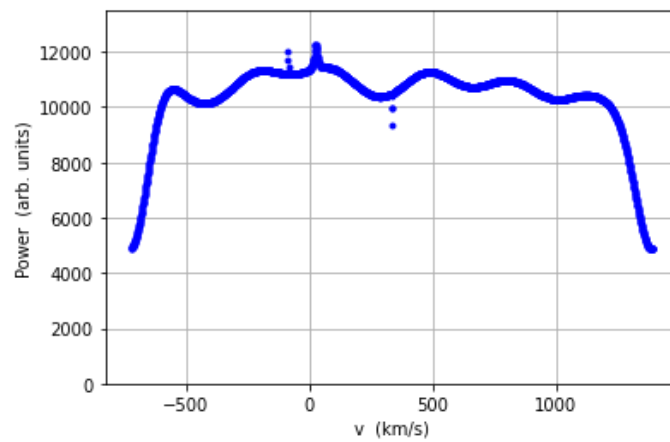


Figure 2. Power intensity produced by different speed rays (credit to Professor Daniel Marlow).

Then, the image is processed to remove the background.

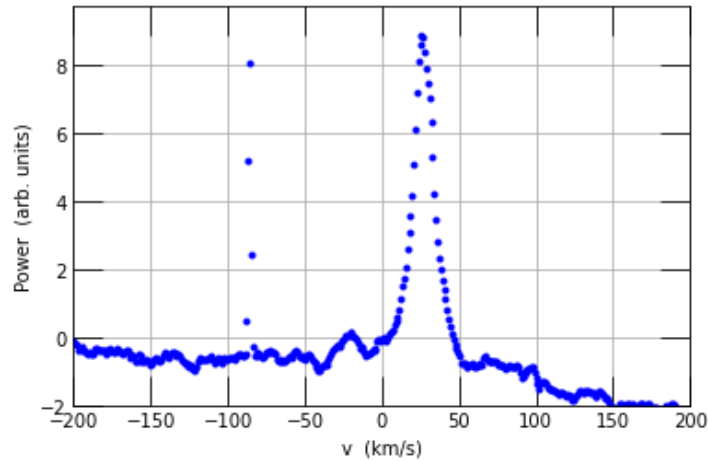


Figure 3. Power corresponding to different speed rays after removing the background (credit to Professor Daniel Marlow).

Get the image of each silver warp, then all the images in this period of time are stacked in order according to the size of silver and latitude.

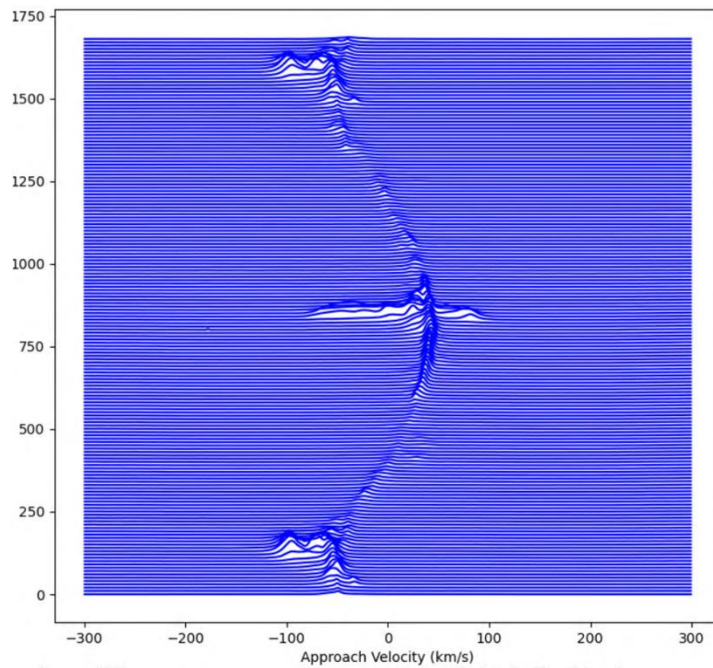


Figure 4. Velocity distribution of received rays at different silver latitudes (credit to Professor Daniel Marlow).

From this picture, we can clearly feel the velocity of the celestial bodies in different positions relative to the earth.

Considering the distance to the center of the galaxy and the unique velocity of the solar system, after some geometric analysis, we can get the relationship between the rotation speed of the inner object and its distance to the center of the galaxy.

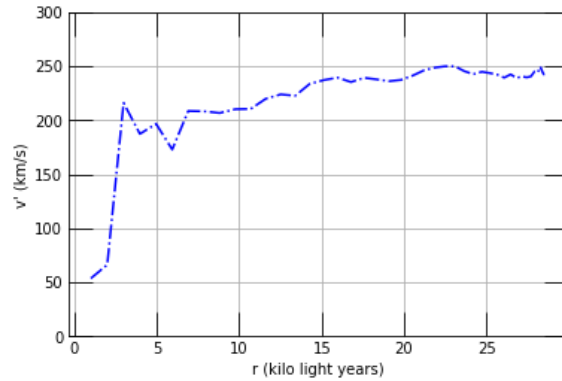


Figure 5. Plot of celestial velocity versus distance to the Silver Center (credit to Professor Daniel Marlow).

2.2. Theory

2.2.1. *Kepler curves.* Through the formula of universal gravitation and the formula of centripetal force, we can simply get the relation between the speed of mass and the distance

$$\sqrt{\frac{GM}{R}} \quad (1)$$

When the M value is constant, the image is in the following form

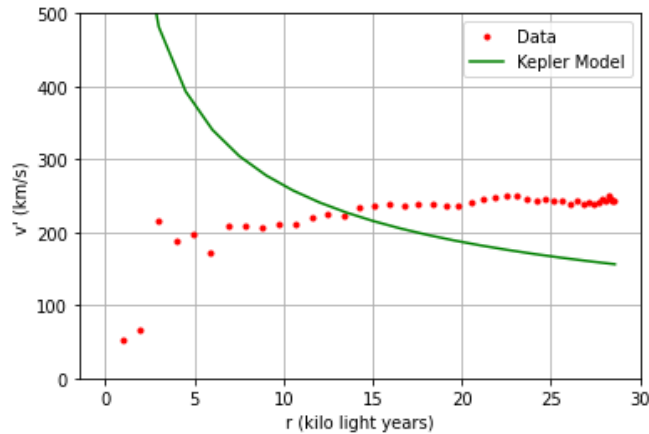


Figure 6. Keplerian curve versus actual curve (credit to Professor Daniel Marlow).

As we can see, the Kepler images don't match the actual observations at all

2.2.2. *Exponential disk model [4].* The reason why the quality is too idealized may lead to the deviation of the image. The distribution of matter in the galaxy is not completely uniform. With this in mind, we need to come up with new theories to refine them. In the Index plate model, the luminosity distribution is considered. The empirical relationship between the luminosity and the mass of a star is

$$\frac{L}{L_{\theta}} = \left(\frac{M}{M_{\theta}}\right)^{\alpha} \quad (2)$$

Among them, L_{θ} refers to the Sun's luminosity, M_{θ} refers to the Sun's mass. Alpha value is 3.5, which means the luminosity positively related to the quality. Celestial luminosity and per unit area to the galactic center distance

$$I(r) = I_0 e^{\frac{-R}{R_D}} \quad (3)$$

At the same time

$$\mu(r) = \mu_0 e^{\frac{-R}{R_D}} \quad (4)$$

Among them, R_D refers to the length of the Milky Way.

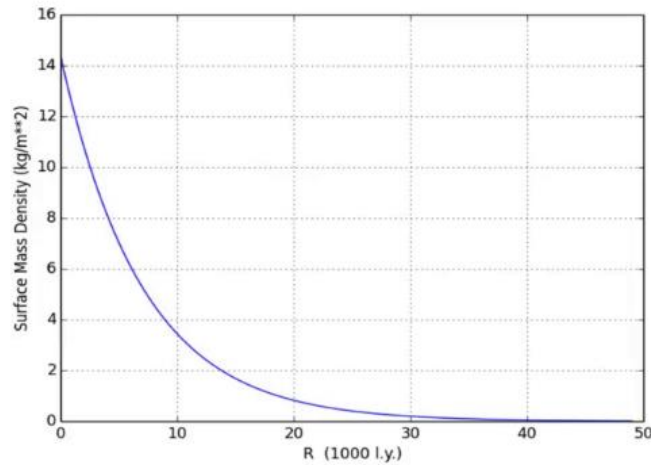


Figure 7. Plot of celestial density versus distance to the silver center (credit to Professor Daniel Marlow).

$$v^2(r) = \frac{GMr^2}{2R_D^3} [I_0(u)K_0(u) - I_1(u)K_1(u)] \quad (5)$$

And we end up with the above function, including $u \equiv \frac{r}{2R_D}$

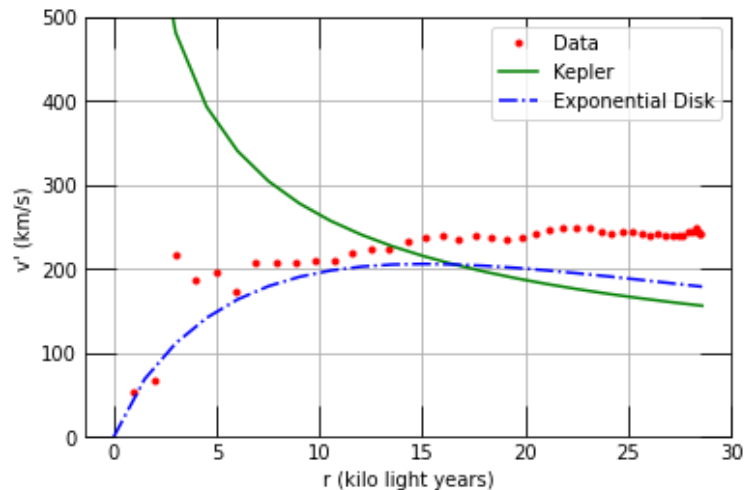


Figure 8. Keplerian curve, exponential disk model vs. actual curve (credit to Professor Daniel Marlow).

However, we found that the index model only matched the data at large trends and positions around 10,000 light-years, while at other times it showed a significant gap. Now that the distribution of mass has been considered, there are two possible explanations for the problem: first, there is still some mass left unaccounted for in the galaxy; second, the law of universal gravitation on large scales is wrong.

2.2.3. *Dark matter halos [5]*. One popular hypothesis right now is the dark matter halo. There is an electrically neutral, non-baryonic, weakly interacting elementary particle in the universe that does not reflect light but has mass. It is the influence of this unknown particle that causes the difference between the actual rotation curve and the index plate model. The dark matter halo hypothesis proposes that this dark matter is widely distributed in the galaxy and affects the speed of celestial bodies [6]. The Dark Matter Halo is assumed to be a spherically symmetric isothermal (the universe is difficult to conduct heat) aggregates, and the following formulas are derived from the calculations

$$\rho(r) = \rho_0 \frac{R^2 + a^2}{r^2 + a^2} \quad (6)$$

Where R is the orbital radius of the solar system and a is the scale factor, ρ_0 is the critical density, and then the mass of dark matter within r is calculated.

$$M_{<}(r) = 4\pi \int_0^r \rho(r')r'^2 dr' = 4\pi\rho_0(R_0^2 + a^2) \left[r - a * \tan^{-1} * \frac{r}{a} \right] \quad (7)$$

Finally, we get the magnitude of the gravitational pull of the dark matter halo.

$$F_h(r) = \frac{GM_{<}(r)m}{r^2} \quad (8)$$

The resulting overall force is the sum of the centripetal forces of ordinary matter and dark matter

$$F_h(r) = \frac{GM < (r)m}{r^2} \quad (9)$$

Eventually, the relation between the velocity and the distance from the whole object to the center of the silver is obtained.

$$v_{tot} = \sqrt{v_{ED}^2 + v_{IS}^2} \quad (10)$$

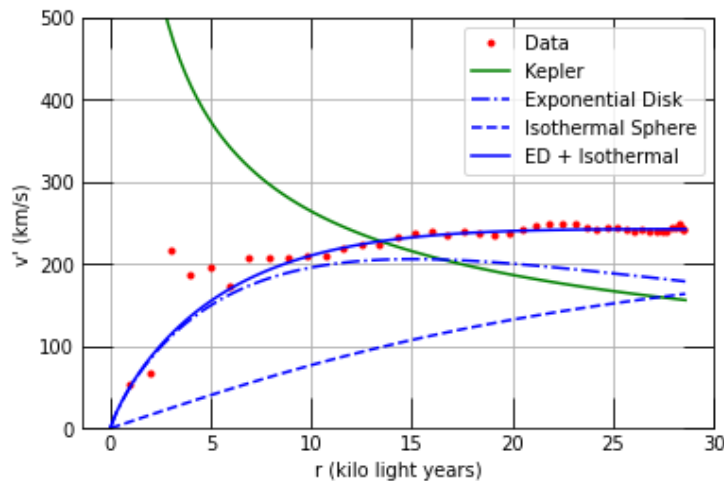


Figure 9. Keplerian curve, exponential disk model, isothermal sphere model, combined exponential disk and isothermal sphere model versus actual curve (credit to Professor Daniel Marlow).

Finally, we get the modified function of adding dark matter and make an image that is very consistent with the actual observations.

But at the same time, another hypothesis, known as MOND [7], argues that Newton's equations of gravity need to be modified, which is Milgrom's law [8].

In the form of

$$m_g \mu\left(\frac{a}{a_0}\right) a = F \quad (11)$$

when $\mu(x \gg 1) \approx 1$, when $\mu(x \ll 1) \approx x$, Where $\mu = x = \frac{a}{a_0}$, a_0 is constant.

This method can also well match the image of the rotation curve of the Milky Way with the observed data, but the theory still faces many troubles, such as the problem of gravitational waves.

3. Conclusions

The development of the galactic rotation curve has undoubtedly had a profound impact on the study of cosmology. From the most ideal, homogeneous mass to the model of the Milky Way to the density of the real Milky Way refracted by luminosity, a basic understanding of what our own galaxy looks like has evolved. The dark matter halo hypothesis and the new MOND hypothesis, both of which have been widely accepted as alternative theories, have also been shown to be plausible by the galactic rotation curve.

References

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