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Simulation of $Pb^{208} + Pb^{208}$ collision using Glauber model

Ning Yan[1*], Bohan Wang[2], Chonglun Xu[3], Xiuqi Qian[4]

[1]Department of Physics and Astronomy, York University, Toronto, M3J 1P3, Canada
[2]AP Department, Beijing National Day School, Beijing, 100039, China
[3]Department of Physics, The University of Hongkong, Hongkong, 999077, China
[4]Department of physics, The University of Manchester, Manchester, M13 9PL, UK

*yn604332298@gmail.com

Abstract. This article is going to discuss several important characteristics of Glauber model from lead isotopes Pb208-under computational analysis. At the start, this paper will provide data under Wood-Saxon distribution in. Then, there are collisions coded from Python, assuming all collisions have reaction cross section of 72 mb, both participant particles and particles under secondary collisions are collected and plot in scatter graphs under impact parameter from 0 fm to 20 fm. Lastly, within the collision area, this paper is going to find the distribution of path length under various parameters.

Keywords: Glauber model, $Pb^{208} + Pb^{208}$ collision, eccentricity, triangularity, computer simulation.

1. Introduction

Glauber model is a physic approximation for multiple particles' collisions made by Roy J.Glauber[1].Glauber was developed this model in 1950's to solve the problem of high energy scattering with composite particles. Glauber further improve his model and present unpublished work in his 1958 lectures [2]. He assumes that all the particles are under high energy acceleration in colliders so that the particles are rarely deviated from their original path and effects such electromagnetic action and multiple action are neglected in these processes under Glauber model. As the developing of supercomputer, the "Glauber Monte Carlo" (GMC) become a new approach to simulate the collision of heavy particle [3]. GMC model the nucleus as uncorrelated nucleons from a measured density distribution. In this paper, the model used is Wood-Saxon distribution. Two independent nuclei will be arranged with impact parameter b and assumed to make collision within to x-y plane. In GMC, the interaction probability can be simulated by the relative distance between two nucleons [4]. Because the elliptic flow signal has been extensively studied in Au+Au collisions, thus this paper will focus on collisions of Pb-Pb collisions in this paper [5].

2. Theory

The GMC this paper used is based on Wood-Saxon distribution, it is a kind of fermi distribution under potential energy in nucleus. The equation predicts [6]:

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$$\rho(r) = \rho_0 \frac{1+w\frac{r^2}{R^2}}{1+e^{\frac{(r-R)}{a}}} \tag{1}$$

Where $\rho(r)$ is the density function of nucleons in Pb 208, ρ_0 is the nucleon density, R is the nuclear radius, w is t。 he deviationns of a spherical shape and a is the skin depth. The value of these parameters can be found in Table 1 [7].

Table 1. Nuclear charge density parameters for Pb [7].

Nucleus	R [fm]	a [fm]	w [fm]
^{207}Pb	6.62	0.546	0

Note: the value for Pb^{208} is close to Pb^{207} because the Bessel-Fourier coefficients for these two nuclei are similar.

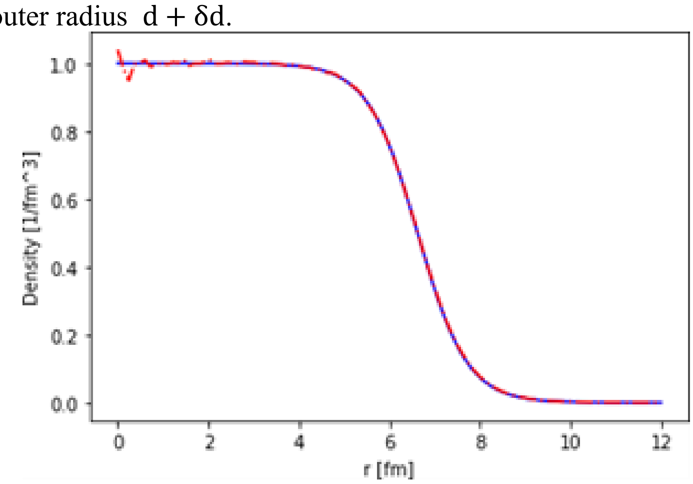
To simplified Eq 1, w=0 can be substitute in and get new version of Eq 1:

$$\rho(r) = \rho_0 \frac{1}{1+e^{\frac{(r-R)}{a}}} \tag{2}$$

This equation can be used to generate the Wood-Saxon distribution forPb^{208}.

3. Generate the distribution

To run the computer simulation of collision, a probability density function of the position for nucleon need be generated from Wood-Saxon distribution. The histogram method is used to generate such distribution [8]. To check the validity of the generated distribution, the random number generated is distributed according to the distance from origin. Then, the density is calculated using the number of random numbers located in distance $[d, d+\delta d]$ from origin divided by the volume of sphere shell has inner radius d and outer radius $d+\delta d$.



Note: Blue line: Wood-Saxon distribution. Red Dashed Line: the generated distribution.

Figure 1. Compare of the generated distribution using histogram method versus Wood-Saxon distribution.

As shown in Figure 1, the density of generated distribution is perfectly fitting Wood-Saxon distribution, which can ideally describe the distribution of nucleon in nucleus. The red dashed line is constructed using 100 million points.

4. Collision

For the collision, two nuclei are assumed to move along z-axis, one is along positive z-axis, and the other is ahead in negative z-axis. The impact parameter b is the distance between center mass of two nuclei. In this paper, b is assumed in x-axis. Thus, the center of two collision nuclei is (-b/2, 0, 0) and

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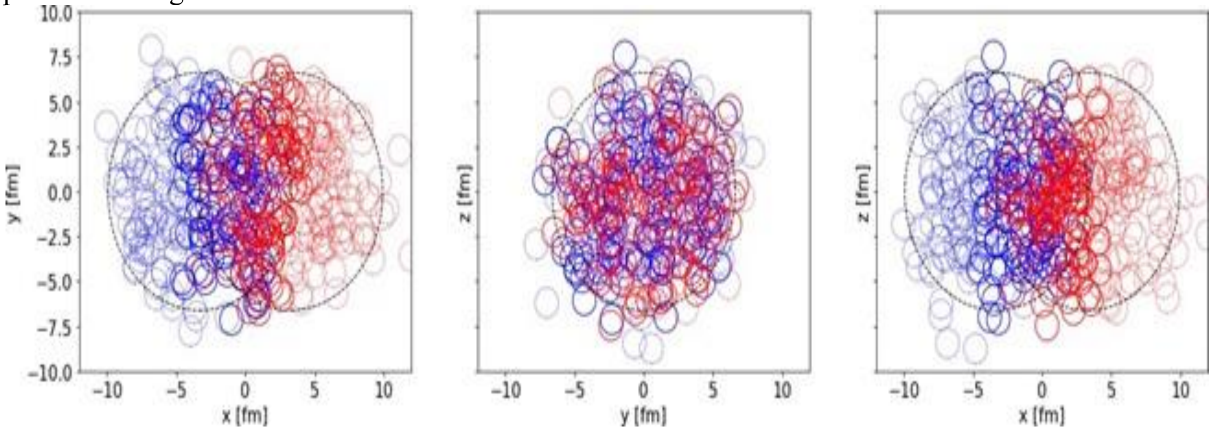
(b/2, 0, 0), If the relative transverse distance of two nucleons is less than ball diameter, then they are collided. The ball diameter is defined as [6]:

$$D = \sqrt{\sigma_{NN}/\pi} \tag{3}$$

Where D is the ball diameter, σ_{NN} is the inelastic nucleon-nucleon cross section and is approximately around $\sigma_{NN} = 72$mb at Large Hadron Collider (LHC). A sample collision of Pb^{208} is provided in Figure 2.



Note: The diagram shows the cross-section of $Pb^{208} + Pb^{208}$ collision. Left: Cross-section in x-y plane. Middle: cross-section in y-z plane. Right: Cross-section in x-z plane.

Figure 2. A sample event for Pb^{208}+Pb^{208} collision.

Green and red circles are participants, and dashed circle are nonparticipants. Left: x-y plane; Middle: y-z plane; Right: x-z plane. The solid circles indicate the participants and dotted circles indicate spectators. The multiple collision event is generated using different impact parameter b. The line connects the center of random generated two collision nuclei may not align with the x-axis. To set b align with x-axis, one should rotate the coordinate system until b perfect align with x-axis. Thus, the impact parameter can be generated by:

$$b = \sqrt{X^2 + Y^2} \tag{4}$$

Where X, Y are uniformly distributed random variable define as the distance of the center nuclei in x, y axis in range [0, 2R]. R is the radius for Pb^{208} nucleus.

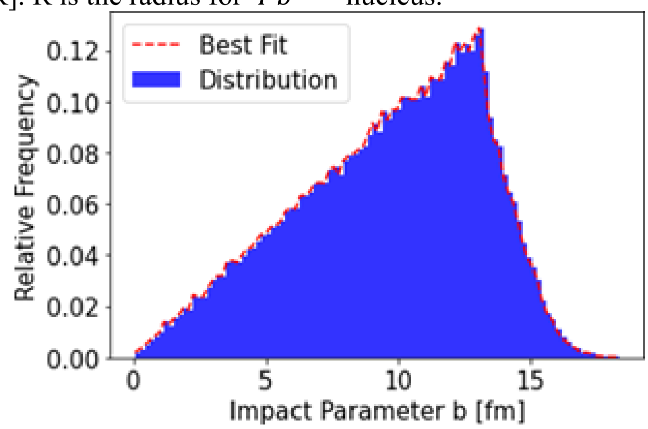


Figure 3. Distribution of the generated impact parameter b.

Figure 3 shown the distribution of generated impact parameter b of 100 million data points. The impact parameter is generated using two uniformly distributed random variables X, Y [0, 2R], where R is set to be 6.62 fm here, and apply to Eq. 4. It is obvious that impact parameter is not likely to be zero.

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The probability is maximum at about $2R = 13.24$ fm. After 13.24 fm, the probability starts to decrease. The distribution appears to close to a bell-shaped but not exact and is right skewed.

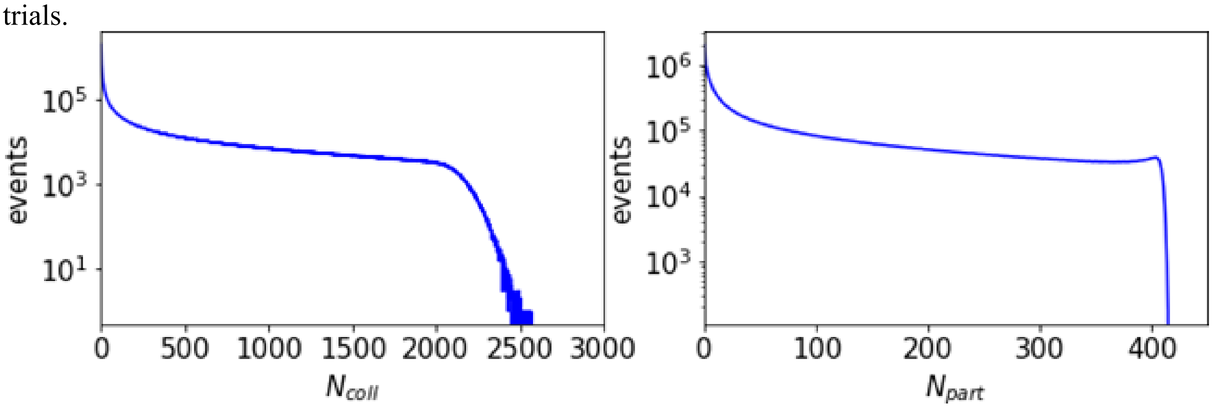
5. Participant

Before discussing the distribution of particles in collision events, it is necessary to define two important parameters first.

N_{part}= the collision participants counted once in the covered areas in range of Pb 208

N_{coll} = the possible collisions occur once the nucleons are within the ball diameter

It is reasonable for N_{coll} to have a much larger scale than the N_{part}, since one nucleon always have many possible collisions with other nucleons. The N_{coll} graph does not have a strict cut but obvious fluctuations, because, in each loop, the random number seed alters once, as said above, the tiny changes in random algorithms cause greater difference in results with the number in algorithms increasing, this phenomenon is greatest at center-to-center collisions. This work has also built up a model to calculate the distribution of the number of collisions and participants in Wood-Saxon distribution 100 million trials.



Note: Left: distribution ofN_{coll}. Right: distribution of N_{part}.

Figure 4. Distribution of N_{part} and N_{coll} for 100 million events.

As the showed in Figure 4, the N_{coll} distribution of both Wood-Saxon distribution has a quick drop near to zero, a flat drop in the middle and a quick drop to zero near to the maximum value. The path length in Glauber model's collision is defined by the number of particles a line meets. The line covers a shape of cylinder with radius of half ball diameter. The graph compares the eccentricity within the collision zone and the angularity within the collision zone of Pb 208 particles' collisions, respectively ϵ_2, ϵ_3.

6. Participant eccentricity and triangularity

Azimuthal anisotropy is a very good tool to study the participant nucleons of the collision nuclei in transverse plane [9,10]. This anisotropy is described by a Fourier expansion of the form [11]:

$$\frac{dN_{part}}{d\phi_{part}} \propto 1 + 2\sum_{n=1}^{\infty} v_n \cos n(\phi_{part} - \psi_n) \tag{5}$$

Where ϕ_{part} is the azimuthal angle, v_n and ψ_n is the magnitude and phase of the n^{th}-order anisotropy of the N_{part}. The eccentricity of the N_{part} denotes as ϵ_2 is given by[12]:

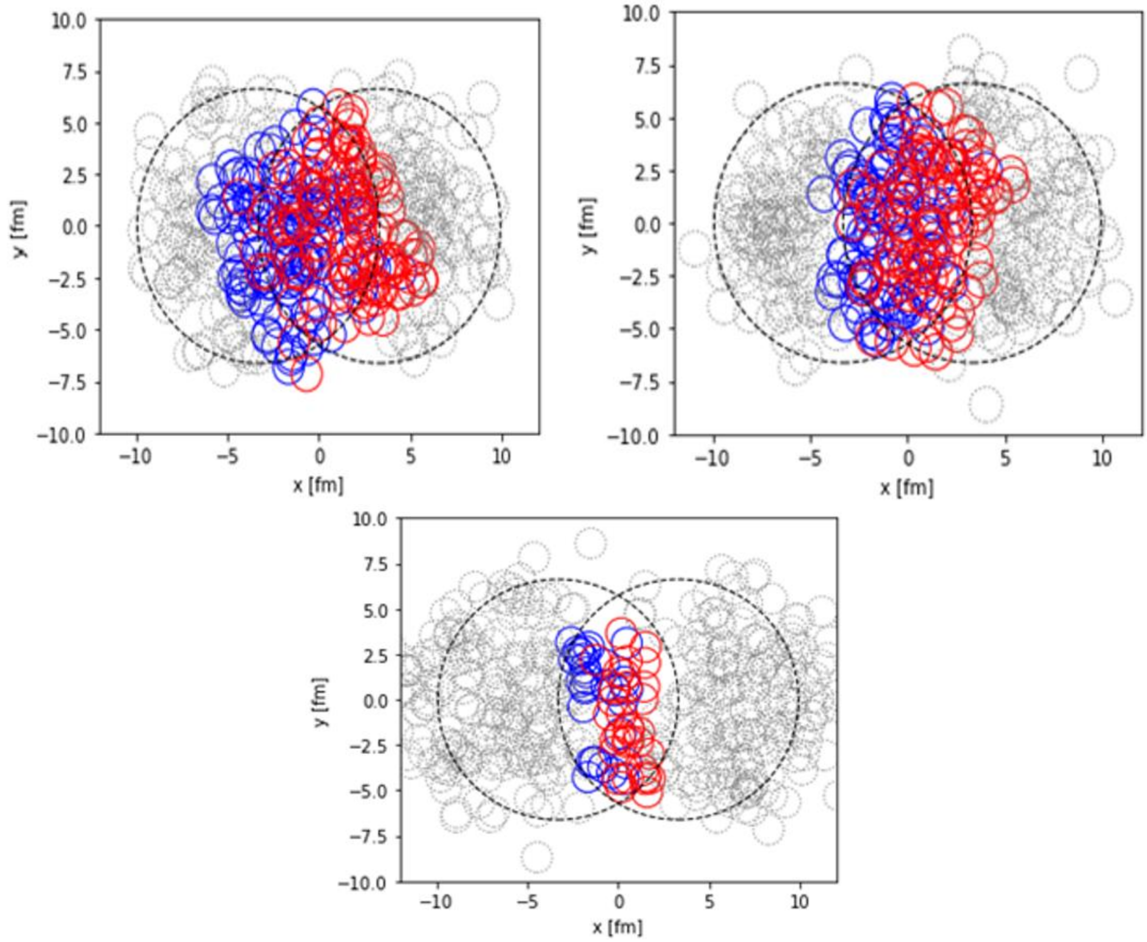
$$\epsilon_2 = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 - \sigma_{xy}^2}}{\sigma_x^2 + \sigma_y^2} \tag{6}$$

It can be shown when the coordinates shift to $\langle x \rangle = 0$ and $\langle y \rangle = 0$ ($\langle\rangle$means the average), Eq 6 is mathematically equivalent to:

$$\epsilon_2 = \frac{\sqrt{\left(\langle r^2 \cos 2\phi_{part}\rangle\right)^2 + \left(\langle r^2 \sin\{2\phi_{part}\rangle\right)^2\}}}{\langle r^2\rangle} \tag{7}$$

The minor axis ψ_2 of the ellipse of N_{part} is given by:

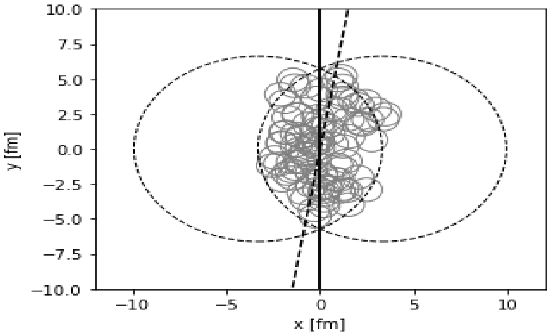
$$\psi_2 = \frac{\mathrm{atan2}\left(\langle r^2 \sin 2\phi_{part}\rangle, \langle r^2 \cos 2\phi_{part}\rangle\right) + \pi}{2} \tag{8}$$



Note: Blue and red circle are participants, and grey dashed circle are non-participants. Top: $\epsilon_2 = 0.02531$, $\psi_2 = 2.9189$, $v_2 = -0.0147$. Middle: $\epsilon_2 = 0.3748$, $\psi_2 = 2.984$, $v_2 = -0.2154$. Bottom: $\epsilon_2 = 0.7066$, $\psi_2 = 0.20979$, $v_2 = -0.5725$.

Figure 5. Collision of Pb^{208}+Pb^{208} collision for differentϵ_2, ψ_2 and v_2 value.

In Figure 5 display the effect of ϵ_2 on the cross-section. Blue and red circle are participants, and grey dashed circle are non-participants. The ϵ_2 can be understood by how similar of the cross-section to an olive shape. The greater ϵ_2 value predicted a better olive shape and vise-versa. As shown in Figure 5, as the ϵ_2 getting larger, the cross-section is more like an olive shape.



Note: The diagram has $\epsilon_2 = 0.5287, \psi_2 = 2.9942$. The grey circle are the participants. The vertical black line is the mirror axis. The dashed black line is the best fit line for the participants. ψ_2 is the angle between mirror axis and the participants best fit line.

Figure 6. Illustration of ψ_2.

Figure 6 illustrate the meaning of ψ_2. The black line is the mirror axis while the dashed line is the best fit line to the participants. The ψ_2 is the angle between mirror axis and the best fit line. Since the participants is olive shaped, which is symmetric along vertical direction with respect to participants. Thus $\psi_2 = 2.9942 = 2\pi - 2.9942 = 0.1474$ Radius.

The magnitude of n^2anisortropy is given by:

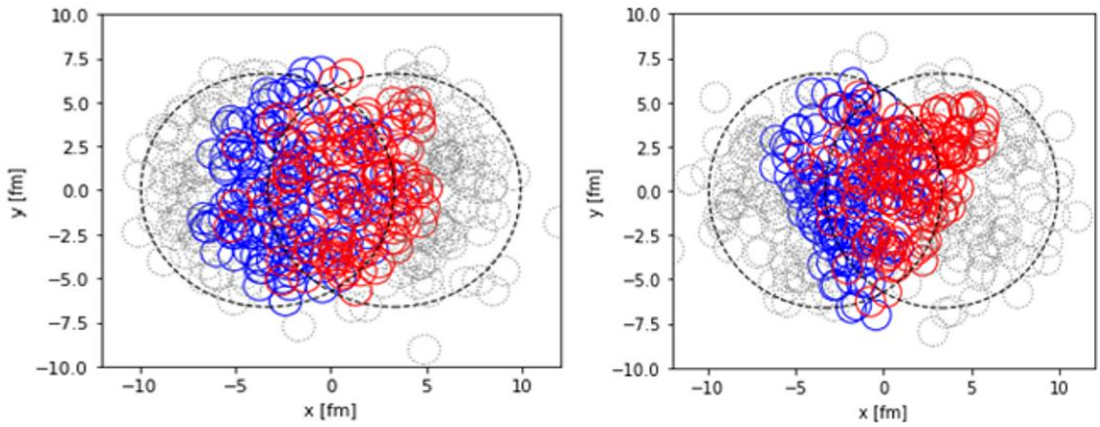
$$v_2 = \langle \cos(2(\phi_{part} - \psi_2)) \rangle \tag{9}$$

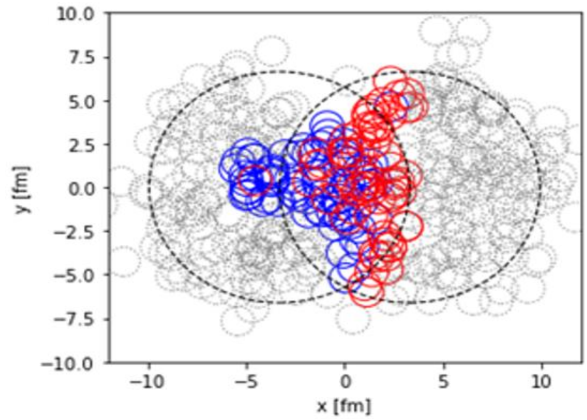
The triangularity of the N_{part} which denotes as ϵ_3 is given by:

$$\epsilon_3 = \frac{\sqrt{\left(\langle r^2 \cos 3\phi_{part} \rangle\right)^2 + \left(\langle r^2 \sin\{3\phi_{part}\} \rangle\right)^2 \}}}{\langle r^2 \rangle} \tag{10}$$

The phase of n^3 anisotropy ψ_3 of the ellipse of N_{part} is given by:

$$\psi_3 = \frac{\mathrm{atan2}(\langle r^2 \sin 3\phi_{part} \rangle, \langle r^2 \cos 3\phi_{part} \rangle) + \pi}{3} \tag{11}$$

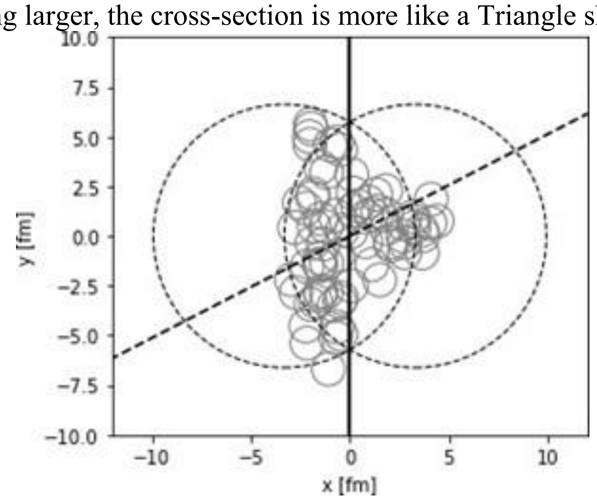




Note: Blue and red circle are participants, and grey dashed circle are non-participants. Top: $\epsilon_3 = 0.0448$, $\psi_3 = 1.143$, $v_3 = -0.0375$. Middle: $\epsilon_3 = 0.3143$, $\psi_3 = 1.471$, $v_3 = -0.1071$. Bottom: $\epsilon_3 = 0.6299$, $\psi_3 = 2.069$, $v_3 = -0.3029$.

Figure 7. Collision of Pb^{208}+Pb^{208} collision for different ϵ_3, ψ_3 and v_3 value.

In Figure 7 display the effect of ϵ_3 on the cross-section. Blue and red circle are participants, and grey dashed circle are non-participants. The ϵ_3 can be understood by how similar of the cross-section to a triangle shape. The greater ϵ_3 value predicted a better olive shape and vise-versa. As shown on Figure 7, as the ϵ_3 getting larger, the cross-section is more like a Triangle shape.

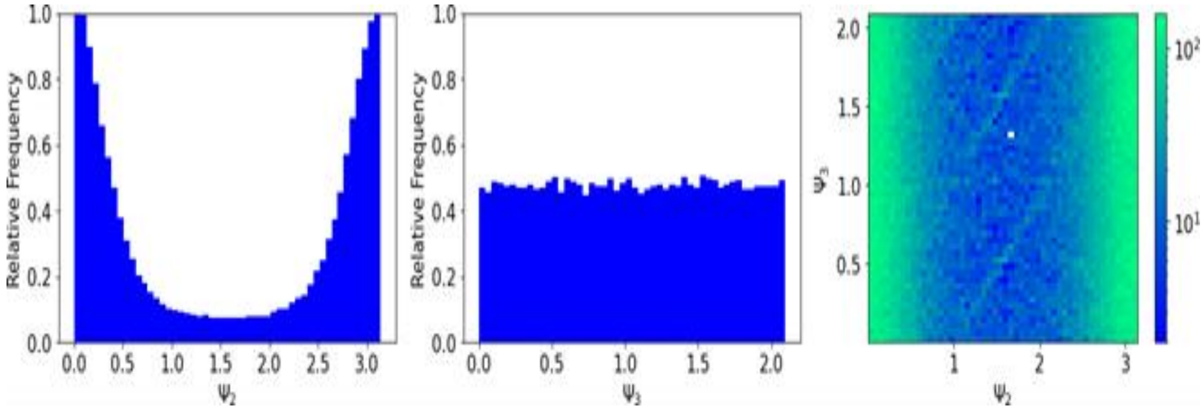


Note: The diagram has $\epsilon_3 = 0.5699$, $\psi_3 = 1.097$. The grey circle are the participants. The vertical black line is the mirror axis. The dashed black line is the best fit line for the participants. ψ_3 is the angle between mirror axis and the participants best fit line.

Figure 8. Illustration of ψ_3.

Figure 8 illustrate the meaning of ψ_3. The black line is the mirror axis while the dashed line is the best fit line to the participants. The ψ_3 is the angle between mirror axis and the best fit line for triangle shape participants. Thus $\psi_3 = 1.097$ Radius. The magnitude of n^3 anisortropy is given by:

$$v\_3 = \langle \cos(3(\phi\_\{part\} - \psi\_3)) \rangle \quad (12)$$

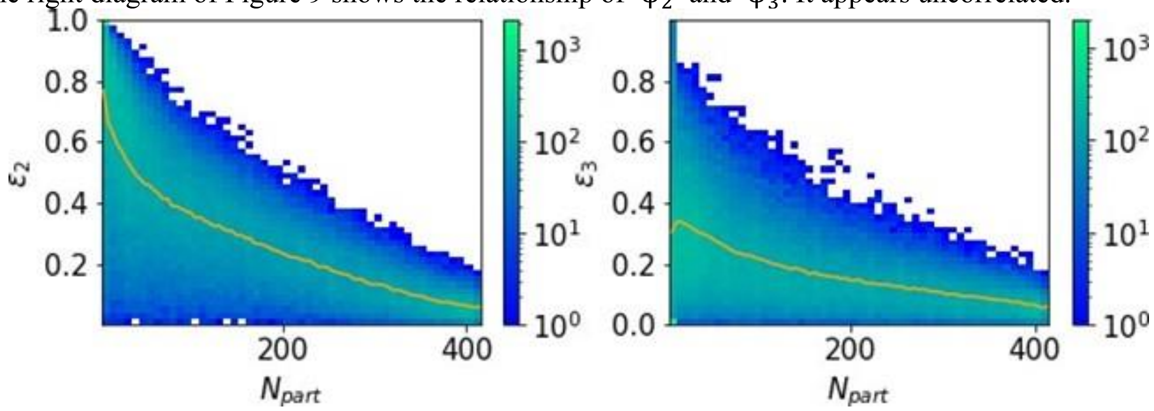


Note: Left: Histogram of ψ_2. Middle: Histogram ofψ_3. Right: distribution of ψ_2 vs ψ_3.

Figure 9. Distribution of ψ_2 and ψ_3.

Figure 9 shows couple distribution of ψ_2 andψ_3. The ψ_2 is heavily distributed around 0 andπ. This is because the shape of the intersection of two collision nuclei is approximately a rugby. And this rugby is point majorly around $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. One intuition way to think about this is put this rugby in complex plane, then the angle is $e^{i\theta}$. When there is the term $\cos 2\theta$ and $\sin 2\theta$, then angle is rotate to $e^{2i\theta}$, which is around 0 and π. Angle ψ_3 is the triangularity flow and it is evenly distributed. This is because the triangle like shape of N_{part} doesn't have a prefer angle in the collision.

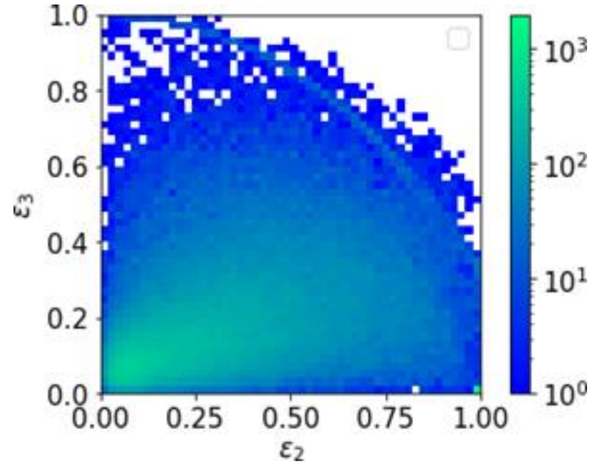
The right diagram of Figure 9 shows the relationship of ψ_2 and ψ_3. It appears uncorrelated.



Note: Left: ϵ_2 vs N_{part}. Right: ϵ_3 vs N_{part}. The yellow line represents the average ϵ_2, ϵ_3 value with respect to N_{part}.

Figure 10. Distribution of eccentricity ϵ_2 and triangularity ψ_3 with respect to participating nucleons N_{part} for Pb^{208}+Pb^{208} collision.

Figure 10 shows the distribution of eccentricity ϵ_2 and triangularity ϵ_3 with respect to N_{part} for Pb^{208}+Pb^{208} collision.



Note: The covariance (ϵ_2, ϵ_3) = 0.008.

Figure 11. distribution of eccentricity ϵ_2 vs triangularity ϵ_3.

Figure 11 shows the distribution of eccentricity ϵ_2 vs triangularityϵ_3.ϵ_2, $\epsilon_3$have a covariance of 0.008, which suggest they are not correlated at all. However, on the diagram, it seems that there is a relationship$\epsilon_2^2 + \epsilon_3^2 \leq 1$. This is an interesting feature and can make further future study.

7. Conclusion

This work applied the Glauber model into Pb^{208}+Pb^{208} collision. This article discussed several important characteristics of Glauber model from lead isotopes Pb^{208}-under computational analysis. A validity approach of generate Wood-Saxon distribution is provided. Meanwhile, the distribution of impact parameter b and participants is provided. Finally, couple feature of participants eccentricity and triangularity is covered.

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