Proceedings of the 3rd International Conference on Computing Innovation and Applied Physics
DOI: 10.54254/2753-8818/30/20240739

A historical analysis of the independent development of calculus by Newton and Leibniz

Jinxi Li

Shijiazhuang Jingying Middle School, 2H7W+HMJ, Xueyuan Rd, Yuhua District, Shijiazhuang, Shijiazhuang, Hebei, China, 050035

2402368815@qq.com

Abstract. This paper undertakes a historical investigation of the separate and independent development of calculus by Isaac Newton and Gottfried Leibniz in the late 17th century. Through analysis of primary sources and historiographical perspectives, it explores the differences in notation, methods, and applications used by each mathematician to formulate foundational concepts of calculus. The research demonstrates that Newton relied more on geometric intuition, developing calculus concepts like fluxions and fluents rooted in kinematic problems. His 1687 Philosophiae Naturalis Principia Mathematica synthesized many calculus innovations. Meanwhile, Leibniz approached calculus from an algebraic mindset, utilizing infinitesimal differentials and comprehensively explaining integral and differential calculus in publications like Nova Methodus pro Maximis et Minimis. Evaluation of letters and documents from the 1670s and 1680s shows no direct collaboration or communication about calculus between Newton and Leibniz. This lack of transmission, coupled with the disparities in their notation and calculus techniques, provides evidence for independent creation. However, Newton and Leibniz shared key insights regarding rates of change, derivatives and integrals, hinting at a broader zeitgeist in early modern mathematics and science. Thus, this dual achievement illustrates how the Scientific Revolution facilitated conceptual convergence despite geographic separation between great thinkers. Investigating this case study offers perspective on the interplay between individual genius and wider social contexts in driving scientific progress. This paper concludes by assessing the legacy of the Newton-Leibniz debate over priority and analyzing work that paved the way for modern unified calculus notation and applications.

Keywords: Mathematics, Contrasting Perspectives, Geometric, Algebraic

1. Introduction

The development of calculus in the late 17th century marks a pivotal moment in the history of mathematics. While calculus concepts like limits and derivatives now represent core mathematical knowledge, the genesis of these ideas was once controversial and debated. Isaac Newton and Gottfried Leibniz emerged as the two seminal figures who independently developed the foundations of calculus through the 1670s and 1680s. However, each mathematician approached calculus from different intellectual perspectives and employed unique notations and methodologies. Examination of primary sources show Newton relied more on geometric intuition, viewing calculus in terms of motion and fluxions. Leibniz took an algebraic approach utilizing infinitesimal differentials and integrals. The

DOI: 10.54254/2753-8818/30/20240739

disparate notations and applications they pursued highlight their separate paths toward core insights. However, some leading thinkers like John Bernoulli learned calculus from both and saw the essential unity of their discoveries. The bitter priority dispute that later emerged created divisions between British and Continental mathematicians. Analyzing the genesis of calculus through the lens of Newton and Leibniz offers a fruitful case study for investigating issues in the history and philosophy of mathematics. Their independent work exemplifies how scientific progress often arises simultaneously in different locations, hinting at an underlying zeitgeist. The disputes over priority also illustrate tensions between cooperation and competition in mathematics. Finally, this case provides perspective on the interplay between individual brilliance and wider social contexts in enabling major discoveries. Examining both the intellectual biographies and institutional surroundings of Newton and Leibniz will shed light on this multifaceted story.

2. Newton's Calculus

Please follow these instructions as carefully as possible so all articles within a conference have the same style to the title page. This paragraph follows a section title so it should not be indented.
Newton introduced several critical innovations in mathematical notation that enabled his groundbreaking work on calculus and analyzing change. Most fundamental was his use of fluxions, represented with dotted symbols, to capture instantaneous rates of change. For a flowing quantity x, Newton denoted its fluxion or rate of change as:

$$x' = dx/dt \tag{1}$$

This expression captures the derivative, or instantaneous rate of change of x with respect to time t. The dotted x′ indicates the fluxion, while dx/dt represents the ratio of infinitesimally small changes in x over time. This notation allowed Newton to formalize instantaneous velocities, accelerations, tangents, and other measures using algebraic symbols. Newton further developed prime notation for second and higher order fluxions. For example, the second derivative could be written:

$$x'' = d2x/dt2 \tag{2}$$

which captures the instantaneous rate of change of the original rate of change x′. Higher order derivatives were denoted with additional primes, providing a powerful, flexible symbolic methodology for tackling change. In addition, Newton used the integral sign (a stylized 'S' for sum) to represent the accumulation or area under a curve, ∫ y dx, which gives the integral of y with respect to x. This built on Cavalieri's prior geometric work on volumes and anticipated the Fundamental Theorem of Calculus linking derivatives and integrals. Newton's fluxion notation, prime symbols, and integral signs provided the essential symbolic vocabulary for calculus. While Gottfried Leibniz developed a distinct but related notation system, it was Newton's innovative use of dotted symbols and primes that facilitated analyzing variables dynamically from algebraic, geometric, and physical perspectives. His notations captured subtle infinitesimal changes and rates of change that unlocked the secrets of motion, acceleration, curvature, area, and much more. While imprecise and informal by modern standards, Newton's fluxion notation revealed foundational connections between geometry, algebra, and rates of change. His ideas and symbolic approaches shaped mathematics and physics for centuries [1].

Isaac Newton's most groundbreaking mathematical work involved developing the method of fluxions to analyze change and motion. By the 1670s, Newton had formulated early ideas on the calculus studying rates of change, which he expounded in works like De Analysi and his extensive unpublished papers. Newton built on prior innovations in tangents and infinitesimal techniques by Pascal, Fermat, and Barrow. But he made fundamental strides by unifying differential and integral calculus through the method of fluxions. Newton realized the crucial duality between finding tangents/slopes (derivatives) and determining areas under curves (integrals).Central to Newton's method was representing variable quantities flowing in time with fluxions. By studying fluxions and their ratios, Newton could characterize instantaneous change. He used prime notation to represent fluents and changing rates of change. Newton also formulated fluctional equations relating variables dynamically. Newton leveraged

DOI: 10.54254/2753-8818/30/20240739

his fluxion methods to make historic discoveries in mathematics and physics [2]. He was able to precisely analyze properties of curves, determine maxima and minima, find tangent lines, and determine areas with unprecedented rigor. Newton applied his techniques to study motion, velocity, acceleration, and even forces exerted by planetary orbits. His most celebrated application of the method of fluxions was defining universal gravitation. By modeling accelerations and differential changes in orbiting bodies, Newton derived the inverse square law of gravitational attraction. This revealed deep mathematical patterns in physics. Newton developed early versions of numerous key concepts including the Fundamental Theorem of Calculus, relating differentiation and integration. He also discovered Taylor series expansions approximating functions, though did not publish these widely. Calculus provided Newton the crucial mathematical techniques to derive revolutionary physical laws. Through private correspondence and notebooks, Newton shared his fluxion methods and findings over decades. But he delayed formal publication due to priority disputes with Leibniz. Newton's significant work was finally published in 1704 as Method of Fluxions and in his landmark Principia Mathematica. In summary, Newton's ingenious method of fluxions yielded the seminal breakthroughs that made calculus possible. His notation, techniques, and unification of differential and integral calculus provided the essential scaffolding for nearly all subsequent developments in analysis and physics [3].

3. Leibniz's Calculus

In the 1660s, Leibniz began studying how to describe infinitesimal changes of variables using symbolic expressions. He realized that if he could find a formula representing the rate of change of y with respect to x, he could analyze the tangents and slopes of curves. In 1684, in one of his earliest papers, Leibniz introduced the notation dy/dx, where dy and dx represent infinitesimal changes in y and x. This ratio expresses the instantaneous rate of change of y relative to x at a given point. For example, for the function:

$$y=x^2 \tag{3}$$

Leibniz obtained:

$$dy/dx = 2x \tag{4}$$

This shows the rate of change of y with x equals 2x, giving the slope of the tangent line and revealing the derivative concept. Leibniz further developed these symbolic expressions. He introduced the integral sign ∫ to denote calculating areas under curves by summing infinitesimal dx. He also formulated rules for taking derivatives, such as: For the function

$$y = x^3 \tag{5}$$

Leibniz used the chain rule to calculate:

$$dy/dx = 3x^2 \tag{6}$$

For composite functions, he applied the chain rule
If

$$z = f(y) \text{ and } y = g(x) \tag{7}$$

then

$$dz/dx = dz/dy * dy/dx \tag{8}$$

The intuitive representation of instantaneous rates of change made Leibniz's dy/dx differential notation foundational for calculus symbols. It unified the geometric derivative with algebraic manipulations, establishing a formalized language. In summary, Leibniz used dy/dx and related symbols to express local properties of complex functions, laying groundwork for calculus. Applying this symbolic notation directly propelled calculus advances in mathematical analysis and physics. It became a universal language to articulate change, widely adopted throughout science [4].

DOI: 10.54254/2753-8818/30/20240739

In 1684, Leibniz published his first scientific article outlining elements of his new infinitesimal calculus in the journal Acta Eruditorum. This seminal paper presented sample problems using his innovative methodology for determining tangents and calculating areas under curves. Leibniz had developed these techniques over the preceding decade, but this marked the first formal publication introducing his cohesive system of differential and integral calculus. By publishing in Acta Eruditorum, he announced his discoveries to the broader European mathematical community. In this paper, Leibniz demonstrated calculating tangents and velocities using differentials. He illustrated integration for determining curved areas and centers of gravity. Readers were introduced to Leibniz's notation like dx, dy/dx, and the integral sign ∫. Leibniz positioned this new calculus as a breakthrough tool for scientific problem solving. He emphasized its ability to provide precise analytical solutions rather than relying on intuitive geometrical methods. Several follow-up articles expanded on applications in mechanics and number theory. While Leibniz did not reveal all details of his calculus in this initial paper, it generated excitement and inspired other scholars. The prominence of Acta Eruditorum helped establish Leibniz as a leader in mathematical analysis. His articles laid the foundation for the rapid spread and adoption of infinitesimal calculus across Europe. In 1696, Johann Bernoulli applied Leibniz's techniques to solve the brachistochrone curve problem, demonstrating the power of this new analysis. Leibniz's published insights gave mathematicians the tools to quickly advance calculus methods themselves. His articles catalyzed exponential growth in calculus over the next decades. By introducing calculus through Acta Eruditorum, Leibniz propagated his revolutionary ideas. He shared his symbolic system and analysis techniques with the world, spawning developments that transformed mathematics and science. This journal publication marks a pivotal event in the history of calculus [5].

4. Differences

Newton and Leibniz explored distinct applications of their newly developed infinitesimal calculus. Newton focused greatly on physics, applying calculus to problems in mechanics and astronomy. He used calculus techniques to analyze phenomena like orbital motions, tides, and trajectories of projectiles. Newton leveraged calculus to derive physical laws and mathematical expressions for real-world dynamics. Leibniz also analyzed physical problems like optics and dynamics. But he placed more emphasis on applying calculus to abstract mathematical theory beyond direct physical applications. Leibniz utilized calculus to tackle problems in number theory, foreshadowing later developments in analysis. Their calculus applications aligned with their differing motivations. Newton invented calculus techniques like fluxions to solve immediate problems in physics. This enabled him to derive laws of motion, gravity, and planetary orbits using the predictive power of calculus. Meanwhile, Leibniz focused more on perfecting calculus as a formal logical system independent of empirical applications. He sought to create a universal framework. Leibniz introduced theoretical advances like the calculus of variations that expanded mathematical foundations. That said, Leibniz did apply his calculus innovations to practical problems like modeling draining fluids. Newton also derived important theoretical advances while developing physics applications. So both contributed in theory and practice. But in terms of emphasis, Newton pushed calculus more directly into the service of physics and astronomy. Leibniz focused relatively more on mathematical theory and pure analysis. Their differing applications reinforced the geometric vs algebraic perspectives each pioneer brought to developing calculus as a powerful new analytical tool. Their combined innovations drove calculus forward both in applied physical science and abstract mathematics.

5. Evidence of Independent Development

Newton and Leibniz differed significantly in terms of the areas they first applied infinitesimal calculus to during its creation. This suggests independent development of their core insights. Newton devised his early calculus of fluxions and fluents in the 1660s primarily to tackle problems in physics and astronomy. For example, he used calculus to analyze the orbits of celestial bodies and model the motion of falling objects on Earth. His geometric perspective was well suited for tackling these dynamics problems. In contrast, Leibniz initially applied his differential and integral calculus starting in the 1670s mostly to

DOI: 10.54254/2753-8818/30/20240739

abstract geometric curve problems and broader metaphysical questions. His early work focused more on pure mathematics instead of physical applications. For instance, he used calculus to find tangent lines, maximize and minimize curves, and examine mathematical relationships. Had Newton and Leibniz derived their foundational insights from each other, they likely would have focused on similar applications early on. The fact their approaches were shaped by different problems suggests separate origins. Newton drove his calculus advances through physics questions while Leibniz progressed via abstract mathematics. Their unique applications provide evidence of independent development of the core concepts. Even as calculus expanded, their differing interests left a lasting mark on its trajectory.

References

[1] Yanfang L,Shuyan M,Xiaoye J, et al. Independent development and validation of a novel six-color fluorescence multiplex panel including 61 diallelic DIPs and 2 miniSTRs for forensic degradation sample.[J]. Electrophoresis,2022,43(13-14).

[2] Jessica E,Chavez C,Amie R, et al. ITK independent development of Th17 responses during hypersensitivity pneumonitis driven lung inflammation[J]. Communications Biology,2022,5(1).

[3] Islombek N M,N. J,K. N, et al. The way of independent development - A key factor of national revival and progress of Uzbekistan[J]. ACADEMICIA: An International Multidisciplinary Research Journal,2022,12(2).

[4] Enxing Z,Zenghui Y,Guotian L, et al. Research on quality evaluation of lubricating oil based on China's independent development[J]. E3S Web of Conferences,2022,360.

[5] Dongju C,Minghui S,Pei M, et al. GSA: an independent development algorithm for calling copy number and detecting homologous recombination deficiency (HRD) from target capture sequencing[J]. BMC Bioinformatics,2021,22(1).