

Chinese chess competition strategy analysis based on the zero-sum game

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Abstract. As a desktop game with a long history, Chinese chess has been widely circulated. In recent years, Chinese chess championships have become increasingly popular. During the competition, in addition to the individual abilities of the participants, the use of different strategies in the competition also become another major factor affecting the outcome of the competition. Based on the fixed ability values of the contestants, this research focuses on discussing the selection of strategies. This paper will adopt points-based match rules in common competitions and quantify the player's ability values for making a data analysis about nine different strategies. Due to the use of a point system in the competition, this research could use zero-sum game for analysis. According to zero-sum game and Nash equilibrium, the payoff matrix could be obtained. Furthermore, a precise linear program model is built. After the calculation, this research presents a result about which strategies should be used in the competition and provide an analytical explanation.

Keywords: Chinese chess competition, zero-sum game, linear programming, "Choking" phenomenon.

1. Introduction

Traditional Chinese chess is a long-standing activity that is well-known throughout the world. It is tied with chess and Go among the world's top three chess sports. It simulates ancient warfare, straight-line warfare, land warfare, and plane warfare, using square chessboards and red and black circular chess pieces to play against each other, with the two sides alternately playing chess, and the opponent's general "checkmate" side wins first. And also, the National Chess Team Championship is very popular now. This type of competition often adopts a rotation system with multiple rounds of competition. This institution determines that team competition is a very strategic competition. The good application of the rotation strategy will bring significant benefits to the team. Therefore, based on the premise that both teams have the same intensity, choosing a good rotation method will have a higher probability of victory for one side.

The system of most team competitions is that the more you win, the higher the probability of winning. But some chess team tournaments use a points system to determine the outcome of the game. In a points format, gain for one party inevitably results in loss for the other. Therefore, it is similar to zero-sum games. Zero sum game is one of the concepts of Game theory, which has been applied in many fields [1]. Specifically, it implies that both sides compete with each other in certain circumstances, and the

sum of their gains and losses will always remain zero. Since the competition adopts the point system, the results of both sides can be quantified, so building the objective function and using linear programming is a good method to solve the two-player Zero sum game [2]. For multi-objective games, Dantzig outlined two methods for examining the existence of weighted Nash equilibrium. The first is the well-known "fixed-point method," and the second is the infrequently used "Ky Fan minimax inequality" in the fields of mathematical programming, game theory, and optimization [3]. In 1990, Patterson and Mike addressed how to turn a bilateral zero-sum game between two players into the linear programming issue and presented a model for computer simulation to solve this problem [4]. And also, the initial approach to developing a mixed 0-1 linear programming for pure-strategy Nash equilibrium was developed by Wu, Dang and Karimi [5]. To be more specific, the primary idea utilized in the analysis of strategic games is Nash equilibrium, which was first introduced in the 1950s. Nash equilibrium is a game in which many people participate, each person develops their optimal strategy based on the strategies of the other players. As long as no one makes any strategic adjustments, at this point, all of the participants' strategies have reached a balance, and this equilibrium is known as the Nash equilibrium [6]. It is also known as non-cooperative game equilibrium since it is primarily used to analyse equilibrium in non-cooperative games. Nash equilibrium's complexity was clarified in 2006 by Daskalakis, Goldberg, and Papadimitriou. They showed that, given the complexity class PPAD, the problem of determining a Nash equilibrium in a game with four or more players is resolved. their proof used ideas from the previously proven equivalence between graphical games and normal-form games in terms of polynomial time solvability [7]. And also, Nash equilibrium could be used in a three-players Zero sum game. Mr. Satoh and Tanaka showed that without the coincidence of the maximin and minimax strategies, an asymmetric equilibrium in a symmetric three-player zero-sum game might exist [8]. Therefore, for points-based games, we can use zero-sum game theory and Nash equilibria to develop an excellent model for creating a better strategy for one of the parties.

This paper's goal is to analyse the best play in this game using a zero-sum game and Nash equilibrium. To achieve this goal, some things used in competitions will be quantified to enhance the accuracy of models. Furthermore, building a precise linear programming model is another purpose. In the end, this essay will determine the best course of action in this chess match.

2. Method

2.1. Data source

To facilitate the establishment of mathematical models and calculations, this paper builds on the rules of the National Chess Team Championship and then quantifies some things in the game, such as the ability of the participants.

2.2. Variable description

There is the chess game set. There are two teams in the competition, Team A and Team B, each consisting of 3 players. the players are ranked from strongest to weakest, denoted as 1, 2, and 3, their abilities are assigned values of 10, 8, and 6 according to their rankings. But if the same player plays two consecutive rounds, his ability will be subtracted by 3. In each round of the competition, both teams select 2 players to participate. A win in a match awards 2 points, a loss awards 0 points, and a draw awards 1 point to each team. X_i $i \in (1, 2, 3, \dots, 9)$ represent 9 different kinds of strategies.

$$\begin{cases} X_1 = (1, 1, 0, 1, 1, 0) \\ X_2 = (1, 1, 0, 1, 0, 1) \\ X_3 = (1, 1, 0, 0, 1, 1) \\ X_4 = (1, 0, 1, 1, 1, 0) \\ X_5 = (1, 0, 1, 1, 0, 1) \\ X_6 = (1, 0, 1, 0, 1, 1) \\ X_7 = (0, 1, 1, 1, 1, 0) \\ X_8 = (0, 1, 1, 1, 0, 1) \\ X_9 = (0, 1, 1, 0, 1, 1) \end{cases} \quad (1)$$

For example, X_1 means two strongest players in two consecutive rounds. X_2 means the two strongest players in round 2, but in round 3 the players are the strongest and the weakest. The followings are the same.

Additionally, the following assumptions are made: if the difference in the total sum of abilities of the two sides is 6 or more, the probability of a draw is 5%; if the difference is 5 or 6, the probability of a draw is 10%; if the difference is 3 or 4, the probability of a draw is 20%; if the difference is 0, the probability of a draw is 40%. The remaining probability to determine the outcome of the game is distributed according to the ratio of the total sum of abilities. However, if the difference in the total sum of abilities is only 1 or 2, the probability of a draw is 30%. Conversely, the weaker team will have a greater chance of causing an upset. It is called the “Choking” phenomenon. The “Choking” phenomenon describes the significant psychological changes that occur in athletes as a result of extreme pressure during competition, which significantly lowers the level of competition and results in anomalous performance. It causes athletes to repeat patterns of sharp drops, which frequently affects the game's outcome much less than anticipated [9, 10].

2.3. Mathematical method introduction

Linear Programming Model: A denotes the payoff matrix, x_0 is objective function.

$$\begin{aligned} & \text{Maximize} && x_0 && (2) \\ & \text{subject to} && A^T x - x_0 \geq 0 && (3) \\ & && x \geq 0 && (4) \end{aligned}$$

Zero sum game: Given a game for two players. In the game, each player receives a reward after making one move concurrently and independently out of a limited number of options. A and B are two real matrices, where a_{ij} represents the prize for Player 1 and b_{ij} represents the prize for Player 2. What one player earns, the other player loses, as in a zero-sum game. It means that $a_{ij} + b_{ij} = 0$.

Minimax Theorem: The Minimax Theorem establishes that a Nash Equilibrium in the sense of mixed strategies always occurs. The following is the theorem.

Let $A = (a_{ij})$ be any $m \times n$ matrix. Then there has a few of probability vectors $x^* = (x_1^*, x_2^*, \dots, x_n^*)$ and $y^* = (y_1^*, y_2^*, \dots, y_m^*)$ such that:

$$\text{If } K(x, y) = \sum_i \sum_j a_{ij} x_i y_j, \text{ then } \min_y \max_x K(x, y) = \max_x \min_y K(x, y).$$

3. Results and discussion

3.1. Descriptive data analysis

The following is the sum of capability values in the nine strategies. The Table 1 provides some intuitive information that strategy x_9 has the lowest sum of capability value which is 16 and strategy x_4 owns the highest sum of capability value which is 25. The ability values of the other seven strategies are not graded, and the numerical differences are not significant.

Table 1. The sum of capability values.

Strategies	Capability values	Distribution of players
x_1	18	(1, 1, 0, 1, 1, 0)
x_2	22	(1, 1, 0, 1, 0, 1)
x_3	20	(1, 1, 0, 0, 1, 1)
x_4	25	(1, 0, 1, 1, 1, 0)
x_5	20	(1, 0, 1, 1, 0, 1)
x_6	24	(1, 0, 1, 0, 1, 1)
x_7	23	(0, 1, 1, 1, 1, 0)
x_8	24	(0, 1, 1, 1, 0, 1)
x_9	16	(0, 1, 1, 0, 1, 1)

By analysing the data in Table 1 and comparing the sum of the capability values of different policies, it is easy to calculate the possibility of draw, winning and losing that one strategy against the other strategies, they correspond to Tables 2, Table 3 and Table 4 respectively.

Table 2. Possibility of draw.

Strategies	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x_1	0.4	0.2	0.3	0.05	0.3	0.1	0.1	0.1	0.3
x_2	0.2	0.4	0.3	0.2	0.3	0.3	0.3	0.3	0.1
x_3	0.3	0.3	0.4	0.1	0.4	0.2	0.2	0.2	0.2
x_4	0.05	0.2	0.1	0.4	0.1	0.3	0.3	0.3	0.05
x_5	0.3	0.3	0.4	0.1	0.4	0.2	0.2	0.2	0.2
x_6	0.1	0.3	0.2	0.3	0.2	0.4	0.3	0.4	0.05
x_7	0.1	0.3	0.2	0.3	0.2	0.3	0.4	0.3	0.05
x_8	0.1	0.3	0.2	0.3	0.2	0.4	0.3	0.4	0.05
x_9	0.3	0.1	0.2	0.05	0.2	0.05	0.05	0.05	0.4

Table 3. Possibility of winning.

Strategies	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x_1	0.30	0.36	0.33	0.40	0.36	0.39	0.40	0.39	0.33
x_2	0.44	0.30	0.33	0.37	0.33	0.37	0.36	0.37	0.52
x_3	0.37	0.37	0.30	0.40	0.30	0.36	0.37	0.36	0.44
x_4	0.55	0.43	0.50	0.30	0.50	0.34	0.34	0.34	0.58
x_5	0.34	0.37	0.30	0.40	0.30	0.36	0.37	0.36	0.44
x_6	0.51	0.33	0.44	0.36	0.44	0.30	0.34	0.30	0.57
x_7	0.50	0.34	0.43	0.36	0.43	0.36	0.30	0.36	0.56
x_8	0.51	0.33	0.44	0.36	0.44	0.30	0.34	0.30	0.57
x_9	0.37	0.38	0.36	0.37	0.36	0.38	0.39	0.33	0.30

Table 4. Possibility of losing.

Strategies	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x_1	0.30	0.44	0.37	0.55	0.34	0.51	0.50	0.51	0.37
x_2	0.36	0.30	0.37	0.43	0.37	0.33	0.34	0.33	0.38
x_3	0.33	0.33	0.30	0.50	0.30	0.44	0.43	0.44	0.36
x_4	0.40	0.37	0.40	0.30	0.40	0.36	0.36	0.36	0.37
x_5	0.36	0.33	0.30	0.50	0.30	0.44	0.43	0.44	0.36
x_6	0.39	0.37	0.36	0.34	0.36	0.30	0.36	0.30	0.38
x_7	0.40	0.36	0.37	0.34	0.37	0.34	0.30	0.34	0.39
x_8	0.39	0.37	0.36	0.34	0.36	0.30	0.36	0.30	0.33
x_9	0.33	0.52	0.44	0.58	0.44	0.57	0.56	0.57	0.30

Table 3 reflects that the disadvantage of strategy x_9 is significant, as it has a lower possibility of winning against the other 8 strategies, which is due to the lower sum of the ability values of strategy x_9 . On the contrary, because of the high sum of capability values of strategy x_4 , its probability of winning against the other 8 strategies is significant highest.

Due to the fact that the points obtained for each lost match are 0, it is not necessary to consider the score and probability of losing the game when calculating the payoff matrix. So according to Table 2 and Table 3, the payoff matrix can be obtained by calculating the expected values. (Each value in the payoff matrix retains two digits after the Decimal separator.) The following represents the payoff matrix.

$$\begin{pmatrix} 0 & -0.08 & 0.04 & -0.15 & 0.04 & -0.13 & -0.11 & -0.13 & -0.04 \\ 0.08 & 0 & -0.03 & -0.05 & -0.03 & 0.03 & 0.02 & 0.03 & 0.14 \\ -0.04 & 0.03 & 0 & -0.1 & 0 & -0.01 & -0.06 & -0.07 & 0.09 \\ 0.15 & 0.05 & 0.1 & 0 & 0.1 & -0.01 & -0.03 & -0.01 & 0.21 \\ -0.04 & 0.03 & 0 & -0.1 & 0 & -0.07 & -0.06 & -0.07 & 0.09 \\ 0.13 & -0.03 & 0.07 & 0.01 & 0.07 & 0 & -0.01 & 0 & 0.19 \\ 0.11 & -0.02 & 0.06 & 0.03 & 0.06 & 0.01 & 0 & 0.01 & 0.17 \\ 0.13 & -0.03 & 0.07 & 0.01 & 0.07 & 0 & -0.01 & 0 & 0.19 \\ 0.04 & -0.14 & -0.09 & -0.21 & -0.09 & -0.19 & -0.17 & -0.19 & 0 \end{pmatrix} \quad (5)$$

The next step is that establish the objective function model and substitute the payoff matrix A into the model for calculation. In this Linear programming model, it is necessary to ensure that only one strategy can be used in the competition, which is the constraint condition of equation (9).

$$\text{Maximize } x_0 \quad (6)$$

$$\text{subject to } A^T x - x_0 \geq 0 \quad (7)$$

$$\sum_{i=1}^9 x_i = 1 \quad (8)$$

$$x_i \geq 0 \quad (9)$$

Then, running the model in Python to get the outcome is the last step. Finally, the outcome is 10 values about $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9$.

3.2. Result prediction and analysis

If only from the perspective of data size, the final result should be that one party in the competition should try to use strategy x_4 as much as possible, while avoiding strategy x_9 . This is because of the sum of the different ability values of the two strategies. Adopting a strategy with higher ability values is natural.

But after the calculation, the result is

$$X = (0, 0.3045, 0, 0.1624, 0, 0, 0.5331, 0, 0) \quad (10)$$

This result shows that the strategies x_2, x_4 and x_8 could be adopted, in particular, strategy x_8 has the largest proportion. To be more specific, in this competition one party will have a greater probability of winning the game when adopting strategies x_2, x_4 and x_8 , especially strategy x_8 .

The final calculated result is a little bit different from the previous prediction, of course, the strategy x_4 is used and strategy x_9 is not applied. The difference is that strategy x_4 does not have a bigger proportion, the reason is that the previous prediction did not consider the setting of "Choking", which is a fascinating part of this paper. It is precisely because of the phenomenon of "Choking" that the competition becomes more complex and unpredictable.

4. Conclusion

This paper presents a data analysis of strategies used in chess competitions. Although some data is quantified instead of existing, they are also as close as possible to real-life competitions. In this research, the linear programming model was created and calculated in accordance with expectations. For results, because of the phenomenon of "Choking", the strategies that should be adopted in the competition will not be directly seen, but the best results will be obtained by using the related principles of zero-sum game and linear programming. After precise calculations, the optimal strategy for one side in this game is to adopt strategies x_2, x_4 and x_8 . Compared to the previously predicted results, this result has some differences, but also some commonalities. Therefore, this result is more in line with the common sense of real-life competitions and can be explained.

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