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Fourier transformation for acoustic: Principle & applications

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Abstract. A The Fourier transform has a wide range of applications in daily life, including physics, signal processing, acoustics etc. The topic of this article is to demonstrate the principle and applications of the Fourier transform in acoustics through theoretical derivation. The paper first derived the basic formula of the Fourier Transform and the related seven theorems. Then the paper detailed the research of Fourier Transform in underwater acoustic pulse signal detection technology. Finally, the application of Fourier Transform in the defect detection algorithm of MEMS acoustic films was detailed. According to the analysis, the paper demonstrated the primary application of Fourier Transform in underwater acoustic pulse signal detection and defect detection algorithm of MEMS acoustic film. Based on the evaluations, this study demonstrated the general application scenario of Fourier Transform and offered theoretical basis of its application in acoustic field, which promotes its developmental potential in the acoustic field. Overall, these results shed light on guiding further exploration of acoustic research.

Keywords: Fourier Transform, principle, applications: acoustics.

1. Introduction

The world’s earliest acoustic research work was mainly embodied in music. The systematic study of acoustics began at the beginning of the 17th century with Galileo’s study of the single pendulum cycle and object vibrations. D’Alangbell first derived the wave equation for strings in1747 and predicted that it could be used for sound waves. Ohm, who found Ohm’s law, proposed in1843 that the human ear can decompose complex sounds into harmonic components and judge audio according to the size of the sound. In the 20th century, due to the development of electronics, the use of electroacoustic transducers and electronic instruments to receive and utilize any frequency, any waveform, almost any intensity of sound waves, has made the scope of acoustic research different. Recent studies suggest that prenatal acoustic signals can program individual phenotypes against predicted postnatal environmental conditions to improve health status [1]. The latest evidence in birds demonstrate that parental vocalisations and sibling embryonic cues can alter individual developmental trajectories post-hatch to cope with particular threats in the environments [2]. The development of modern acoustics also greatly improves people’s living standard. When people take plane, the internal noise pleasance is important to achieve the optimal acoustic design, and there are several studies obtaining the significant difference of perception by understanding the aircraft internal noise characteristic perception. According to this feature, the body and the transmission can be coupled in detail to obtain effective noise control measures [3].

The Fourier transform has made extensive contributions in acoustics. To be precise, the Fourier transform is the seemingly chaotic signal into a certain amplitude, phase, frequency of the basic sinusoid (cosine) signal combination. The purpose of the Fourier transform is to find out the basic sinusoid (cosine) large amplitude (high energy) signal's corresponding frequency and chaotic signal of the main vibration frequency characteristics. For example, people make experiments of the surging sound signal of compressor, and further use the fast Fourier transform spectrum analysis of experimental data, to represent the characteristics of the compressor into the surge sound signal, which provides a good theoretical basis and evidence for the actual production of the sound signal monitoring compressors status and fault diagnosis; A measurement and analytical method has also been developed by Fourier analysis, which is capable of detecting errors and faults in the interior of machine tools and in the manufacturing process using acoustic sensors (microphones) [4]. In terms of acoustics from the latest study, the piano tone recognition and electronic synthesis system based on Fourier Analysis can be used to analyse the functional modules of the system in detail. Under this method, the experiments concluded that at the frequency of 600-2100Hz, the piano tone recognition and electronic synthesis system remain in a stable state, with good performance, and the timbre recognition is high [5]. Some researches implementing fractional Fourier transform and solving partial differential equation using acoustic computational metamaterials in space domain, which promotes the design theory of acoustic computing metamaterial resigned acquisition and processing, network computing, and facilitate the new application of acoustic waves [6]. In car fields, the starting auxiliary function effectively solve the problem of insufficient torque when the small displacement engine vehicle starts, but at the same time, it causes a large roar in the car. Short-time Fourier transform analysis method can be used to extract the feature frequency to analyse following the vibrational noise performance development principle.

This article aims to take a general summary on the basis of the original Fourier analysis theory. First, the basis format of Fourier transform will be introduced, and the article applied it to different fields of acoustics, then the general idea is presented with "applied scenario—basic equation—presentations of some results".

2. Basic formula of fourier transform

Assuming that f(x) and g(x) are continuous over R and have a first-order continuous derivative. The following formulas are the basic expressions for the Fourier transform,

$$g(x) = \int_{-\infty}^{\infty} G(f) e^{j2\pi f x} df \tag{1}$$

$$G(f) = \int_{-\infty}^{\infty} g(x) e^{-j2\pi f x} dx \tag{2}$$

where the two integrations are called Fourier Integrations, G(f) is the Fourier Transform or frequency spectrum of g(x). If g(x) is a physical quantity indicating certain physical domain, G(f) is the expression of g(x) in frequency domain. G(f) acts as the similar role of Fourier coefficient cn, namely weight factors for various frequency components, which describes the relative amplitude and phase shift of each complex exponential component. When G(f) is complex function, the transform can be described as:

$$G(f) = A(f) e^{j\phi(f)} \tag{3}$$

where A(f) is the amplitude spectrum of g(x); $\phi(f)$is the phase spectrum of g(x). Sometimes simplified marks are used to describe the relationship between g(x) and G(f) as $F\{\cdot\}$. The two-dimensional Fourier transform is a generalization of the one-dimensional Fourier transform:

$$g(x,y) = \iint G(f_x, f_y) \exp\left[j2\pi(xf_x + yf_y)\right] df_x df_y = F^{-1}\{G(f_x, f_y)\} \tag{4}$$

$$G(f_x, f_y) = \iint g(x,y) \exp\left[-j2\pi(xf_x + yf_y)\right] df_x df_y = F\{g(x,y)\} \tag{5}$$

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When the Fourier integral makes sense, it should satisfy following three conditions:
(1) g(x, y) is absolutely integrable throughout the entire x-y plane.
(2) In any limited area, g(x) must have a limited break and a limited maximum and minimum points.
(3) g(x) must have no infinite discontinuity.

2.1. *The principle of Fourier Transform*

Supposing $G(f_x, f_y)$ and $H(f_x, f_y)$ are the Fourier Transforms of g(x, y) and h(x, y). Therefore, several principles of Fourier Transform can be given below. For Linearity Principle,

$$F\{\alpha g + \beta h\} = \alpha G + \beta H \tag{6}$$

which namely the transform of the sum of the two functions is equal to the sum of their respective transforms. As for similarity principle:

$$F\{g(ax, by)\} = \frac{1}{|ab|} G\left(\frac{f_x}{a}, \frac{f_y}{b}\right) \tag{7}$$

Namely the expansion of coordinate x, y in physical space causes the compression of f_x, f_y. As for the displacement theorem:

$$F\{g(x-a, y-b)\} = G(f_x, f_y)\exp\left[-j2\pi(f_x a + f_y b)\right] \tag{8}$$

Namely the displacement of functions in physical space causes linear phase shift of functions in frequency space. As for Parseval theorem:

$$\iint |g(x,y)|^2 dxdy = \iint \left|G\left(f_x, f_y\right)\right|^2 df_x df_y \tag{9}$$

If $g(x, y)$ is an actual physical signal, $\left|G\left(f_x, f_y\right)\right|^2$ is often called energy spectrum. This theorem indicates that the energy of the signal in the physical domain is conserved with its energy in the frequency domain. As for convolution theorem [7]:

$$F(g, h) = G * H \tag{10}$$

When a complicated function can be demonstrated as the multiplication or convolution of simple functions, the transformation formula of complicated functions can be figured out through Convolution Theorem. And the theorem offers another access to obtain the convolution of two functions by multiplying the two transformation formulas of two functions and taking inverse transformation of the multiplication.

2.2. *The Fourier-Bessel transformation*

Given a function g in polar coordinates, if its variables are only the radius r, which means

$$g(r, \theta) = g_R(r) \tag{11}$$

Then, one calls it circumsymmetric. The Fourier Transform of g will be changed to

$$G(\rho, \phi) = \int_0^\infty r g_R(r) dr \int_0^{2\pi} \exp\left(-j2\pi r\rho\cos\left(\theta - \phi\right)\right) \tag{12}$$

Through the Bessel identity [8]:

$$J_0(a) = \frac{1}{2\pi}\int_0^{2\pi} \exp[-jacos(\theta - \phi)]\, d\theta \tag{13}$$

where J_0 is the first class of Bessel function of zero order, substitute it into formula (12), it turns into

$$G(\rho) = 2\pi \int_0^{\infty} r g_R(r) J_0(2\pi r\rho) dr \tag{14}$$

As transformation formula doesn't rely on ϕ and only depends on ρ, then one can replace $G(\rho, \phi)$ with $G(\rho)$. It means that the Fourier transform formula of circular symmetric function is also itself circularly symmetric, it can be obtained by one-dimension calculation. One calls the special form of Fourier Transform as The Fourier Bessel Transform. It can be proved in a similar way that the inverse transform of circusymmetric function $G(\rho)$ is $g_R(r) = B^{-1}\{G(\rho)\}$.Therefore, there is no difference for circusymmetric function between transform and inverse transform. The Fourier-Bessel Transform is a special circumstance of circusymmetric function in two-dimensional way. So, the relevant principles about Fourier Transform are completely suitable for Fourier-Bessel Transform, but the form of expressions may vary. For example, Similarity Principle turns into

$$B\{g_R(ar)\} = \frac{1}{a^2} G\left(\frac{\rho}{a}\right) \tag{15}$$

3. Underwater acoustic pulse signal detection

The presence or absence of a signal is the first and most important step in signal processing. Most of the current classical algorithms are used for the detection of single-frequency signals, and the detection performance of time-varying signals often fails to meet the needs of practical applications. With the development of sonar technology and underwater acoustic engineering technology, human continuously strengthen Marine resources exploration, Marine biological survey, underwater acoustic target detection and underwater acoustic communication, etc. Underwater sonar platform often received a variety of sound signal to detect the signal in order to further obtain the information in the signal. A skecth if shown in Fig. 1. However, for the commonly used blind detection techniques of single-frequency and frequency modulation signals with unknown parameters, in order to improve the overall performance of detecting different types of signals to noise ratio, fractional Fourier transform can help with the blind detection techniques of underwater weak signals [9].

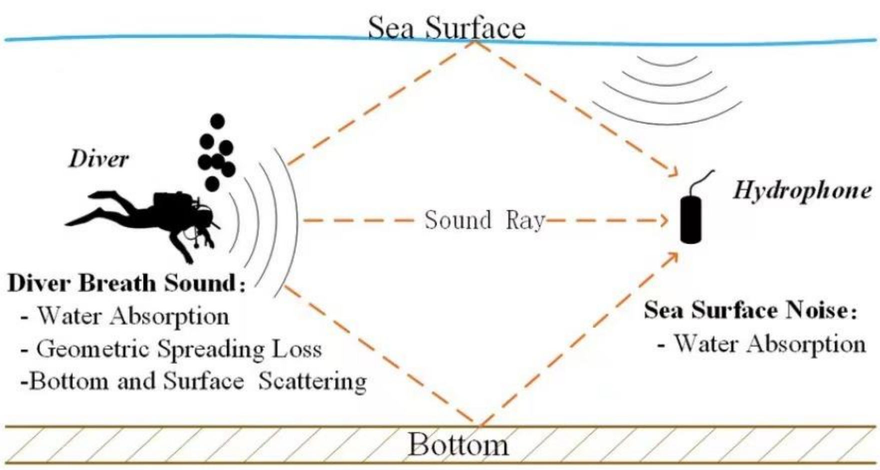

Figure 1. Research on underwater acoustic pulse signal detection.

As a matter of fact, fractional Fourier transform can also be understood as the decomposition of chirp base, fractional Fourier transform of different order reflects the projection of function frequency on the reversed time-frequency axis, because the inverse transform of Fourier transform is

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$$x(t) = F^{-p}\left[X_p\right](t) \tag{16}$$

where X_p means the fractional Fourier transform of antiderivative in p order. It can be inferred that in the above equation that the basis function of the fractional Fourier transform is a linear FM function, and the amplitude of the fractional Fourier transform will reach the maximum when the frequency modulation slope of the basis function is the same as the acoustic signal. When we substitute some data, supposing that the frequency of sampling is 500Hz, the central frequency is 100Hz, tape span is 100Hz. The result is that linear FM got the peak at a p of 0.87, the position of the peak is correlated with the FM slope and phase of the signal, and its amplitude is correlated with the power of the signal. The specific relationship is

$$\{\alpha_0, u_0\} = argmax|X_\alpha(u)|^2 \tag{17}$$

With

$$\begin{cases} \mu_0 = -cot\alpha_0 \\ f_0 = ucsc\alpha_0 \\ \phi_0 = \arg\left[\frac{X_{\alpha_0}(u_0)}{A_{\alpha_0}\exp\left[j\pi u_0^2 cot\alpha_0\right)\right]}\right] \\ \alpha_0 = \frac{\left|X_{\alpha_0}(u_0)\right|}{\Delta t|A_{\alpha_0}|} \end{cases} \tag{18}$$

Here, μ_0 is the frequency modulation, f_0 is central frequency, ϕ_0 is the phase, α_0 is the amplitude. When the target acoustic signal is the FM signal, the function will form a maximum peak on the fractional order of Fourier transform of the corresponding order. By entering the peak detection, the existence of the acoustic signal can be judged.

4. Defect detection algorithm of MEMS acoustic film

The micro-electro mechanical system (MEMS) acoustic film has tremendous demand for tape-out, storage and packaging environments, and its surface defects may have an impact on the quality and performance of MEMS devices [9]. The image defects detection is an effective non-contact optical detection means that can effectively improve the yield rate of MEMS production. However, the periodic structure texture of the MEMS affects defect detection. Therefore, a proposal of an acoustic film defect detection algorithm based on devices surface can be formed. Through the calculation of gradient distribution of spectrogram and establishment of the Booleam mask, the dominant frequency components in accordance with the periodic structure texture were abandoned. The reconstructed images were decomposed by single-layer Haar wavelet to obtain the low-frequency sub-hand image and the defect information was extracted by simple threshold segmentation. The defect detection effects of different types of MEMS acoustic film could be demonstrated (seen from Fig. 2).

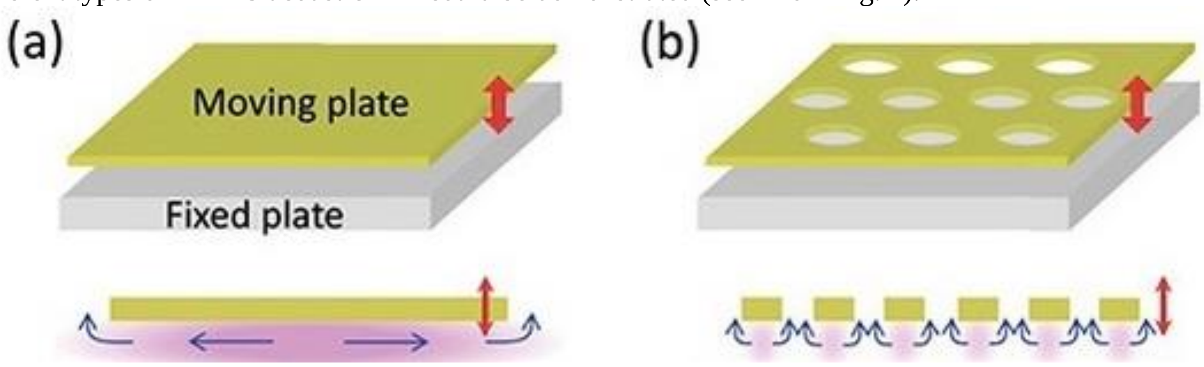

Figure 2. Development of MEMS acoustic film.

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The denoising principle of wavelet transform is to decompose the noise-containing signals into different scales [10], and then reconstruct the wavelet coefficient through the difference of wavelet coefficient of noise signal and effective signal, so as to reduce the influence of noise. The specific steps are listed as follows. Noise-containing signal $f(t)$ can be expressed as $f(t) = s(t) + \sigma e(t)$, where s(t) is actual effective signal; e(t) is noise signal; σ is noise coefficient; M is the sampling sequence length of f(t). For further analysis, the signal f(t) containing noise is expressed as superposition of low frequency and high frequency signals. The wavelet coefficients are reconstructed by thresholding. This algorithm includes the henursure hard threshold algorithm, heursure soft threshold algorithm and the sqtwolog fixed-thresholding algorithm. The principle of heursure hard threshold algorithm is to compare the wavelet coefficients with the threshold value, when the kth coefficient on the j storey $\omega_{j,k}$ is less than the threshold, it can be inferred that judge coefficient was caused by acoustic noise, then it will be put back to 0; when $\omega_{j,k}$ is bigger than threshold, judge coefficient was caused mainly by useful signal. In terms of the elimination of periodic background texture, supposing

$$F(u,v) = \sum_{X=0}^{M-1} \sum_{Y=0}^{N-1} f(x,y) e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})} \tag{19}$$

The amplitude of $F(x,y)$ has the definition $A(u,v) = |F(u,v)|$. Usually, it has a wider dynamic range. However, the gradient value of high frequency is much lower than the gradient of low-frequency area. Therefore, the log spectrum method can be adopted to reduce the range of value:

$$L(u,v) = \lg[A(u,v) + 1] \tag{20}$$

The calculation of gradient value conforms to the following definition:

$$\nabla L = \begin{bmatrix} g_u \\ g_v \end{bmatrix} = \begin{bmatrix} \frac{\delta L}{\delta u} \\ \frac{\delta L}{\delta v} \end{bmatrix} \tag{21}$$

Then gradient diagram can be described according to each gradient value of (u, v)

$$G(u,v) = mag(\nabla L) = \sqrt{g_u^2 + g_v^2} \tag{22}$$

5. Limitations & prospects

Firstly, limited by space, the paper introduces few types of application of Fourier transform in acoustics, which makes the universality of the article confined to some extent. Secondly, despite of text description, there is some hopping in the derivation between formulas, which requires a higher mathematical threshold of readers. Thirdly, in the derivation of the Fourier Transform, the continuity, derivativity, differentiation and other basic mathematical properties of the function have been assumed naturally. This natural preset may catch readers off guard.

The Fourier Transform analyses the functions to achieve a deep understanding and study of the complex functions. As a tool for studying the analytical analysis of thermal processed, the thought approach of the Fourier transform is typical of reductionism and analytics. “Arbitrary” functions, through a certain decomposition, can represent the form of linear combinations of sine functions, which are physically well-studied and relatively simple classes of functions. Using this, the Fourier Transform can understand complex things through the study of relatively simple things. On this basis, the Fourier Transform has a promising future prospect. In many electroacoustic parts, equipment, electroacoustic system and acoustic system, electroacoustic measurement and analysis are often needed to be carried out [11]. Through the measurement and analysis, its acoustic properties can be understood, the electroacoustic technical indicators can be mastered, the system performance can be improved and perfected and the system structure can be optimized. In the meantime, the problems of interconversion between time domain signals changing with time and frequency are often encountered. In order to complete this transformation, fast Fourier Transform is widely used, which is convenient and effective, providing great convenience for recording data analysis.

The Fourier transform is also widely used in spectrum detection and application. As the spectrum detection of electronic devices is limited by electronic bottlenecks, people's growing needs cannot be met in time, which leads to the emergence of radio frequency, frequency hopping communication and various new technologies. Spectrum detection based on optical Fourier Transform realizes a wide variety of detection schemes with the help of the high precision of microwave technology and the speed of photon technology. However, the high resolution in this scheme requires a large dispersion. Therefore, a second utilization of the Fourier Transform can overcome limitations such as lower resolution and smaller bandwidth, which brings a bright future prospect. In case of cutting-edge technology, optical signal processing has been extensively studied over the last half century, its basis is the lensed Fourier Transform effect [12]. The effect is completed in an instant when the light wave travels. The processing speed is irrelevant with the information of the processed signal such as pictures and sizes, classical applications include low-pass, high-pass filtering, convolution, deconvolution, and the correlation identification of graphs. In recent years, the computing function of electronic signal processor has increased rapidly, and the superiority of optical signal processing is gradually manifested. Optical Fourier Transform in some applications are still unable to be replaced with electronics method, such as three-dimensional holography and holographic storage. In fact, the application scope of Fourier Transform is beyond the category of two-dimensional image and signal processing, for example, spatial coherence function can be obtained by the light source intensity distribution of Fourier Transform, computing tomogram can be reconstructed through Fourier Transform. In addition, the spectrum can be through the Fourier Transform spectrometer data processing and then an interesting phenomenon can be got that the inverse processing of the Fourier Transform spectrometer process is implemented as well as the light source wavelength scanning, optical fiber distribution sensor. Fourier transform can also be used to explain the near field optical phenomenon, the near optical image or "nano" linear degree image processing.

6. Conclusion

In summary, this study describes the basic formula of the Fourier Transform and its partial application in the field of acoustics. The paper first derived the basic formula of the Fourier Transform and the related seven theorems. Then the paper detailed the research of Fourier Transform in underwater acoustic pulse signal detection technology. Finally, the application of Fourier Transform in the defect detection algorithm of MEMS acoustic films was detailed. The applications cited in this paper have a small number of applications in the field of acoustics and cannot have a high universality. Overall, the flexible freedom of the Fourier Transform gives it great potential in the future. These results demonstrated the general application scenario of Fourier Transform and offered theoretical basis of its application in acoustic field, which promotes its developmental potential in the acoustic field.

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