Proceedings of the 2023 International Conference on Mathematical Physics and Computational Simulation
DOI: 10.54254/2753-8818/28/20230406

The principle and state-of-art applications of Gravitational lensing

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Abstract. Gravitational lensing, since Einstein proposed, has developed into an essential part of astronomical exploration. On this basis, lots of projects will observe high-intensity explosive transients, such as gravitational waves generated by the merging of dense binaries, so as to improve the accuracy of cosmological observation. Strong gravitational lensing is an effective observation method, which can be used to observe dark mass (sub) halos and test various dark matter models. This study takes Gravitational wave as an example, analyses the application background and mode of explosive transient, and introduces the new measurement methods for dark matter detection. The measurement of redshifted chirp mass and luminosity distance of Gravitational wave is introduced through the formula. Analysis of the characteristics of dark of the pertuber through more group detections will be helpful for new investigation method in the future. In order to make a credible judgment of the nature of the universe, more experiments need to be carried out in these two aspects. This study briefly analyses the progress in these two fields, aiming to encourage more future exploration.

Keywords: gravitational lensing, explosive transients, Gravitational wave, dark matter.

1. Introduction

At present, it is widely recognized that gravitational lens is a major astronomical phenomenon caused by general relativity, i.e., when a huge matter causes gravity, the matter behind it will deflect the light [1]. According to Einstein's research records, which began in 1912, he proposed a theory of gravitational lens three years later [2]. In 1936, Einstein published a short paper in the journal Science under the name "Lens-Like Action of a Star by the Deviation of Light in the Gravitational Field". For some of the effects of gravitational lenses, Einstein only put forward some equations without any deduction. Since he does not have much time to observe this phenomenon, the investigation is not sufficient. In 1959, it has been first shown that when a ray passes through a very close object, such as a black hole, a greatly deviated ray bypasses several circles and escapes, followed by a relative image on both sides of the object. [3].

Since then, with the scarcity of celestial observation data and the complexity of observation data, gravitational lens analysis technology based on deep learning has gradually sprung up. Gravitational lensing has three regimes: strong, weak, and micro. In a strong lensing, the light source, lens, and observer are closely aligned, and the light source is lensed into multiple images. When the background source expands in space, the image is strongly distorted into a circular/curved shape. In a weak lensing, the light source is further away from the line connecting the observer and the lens. Therefore, there is only weak distortion from a single imaging background source. The small distortion of the image caused by weak lenses is usually not obvious to the observer's eye, as the distortion in the image is smaller than the inherent shape of the source. In a micro lensing, when a star is aligned between the observer and the background light source, multiple images of the light source are generated. However, the interval between multiple images is on the microsecond level. This image splitting at the microsecond scale is indistinguishable by telescopes. The observer only needs to see the brightening of the background light source, as the lens forms multiple unresolved images as it passes between the observer and the light source [4]. Recent astronomical studies have shown that in this process, the space is expanding rapidly at a rate of about 30%, while dark matter and dark energy account for about 30% and 70% of the total space, respectively. Furthermore, strong gravitational lensing is helpful to limit dark matter model.

This paper will firstly introduce the basic principle of Gravitational lens, including lens equation, deflection angle and time delay. Subsequently, this study will discuss and analyse the application direction of Gravitational lens in recent years. The conclusion will be given after analysing the current difficulties and future prospects of Gravitational lensing.

2. Theory

Light from a single light source will be transferred by cosmic objects that are farther, larger, or more compact than the sun. The gravity of these objects is strong enough to allow light to approach the observer after transferring multiple rays. The source appears multiplied by the image because the observer sees an image in the direction of every ray approaching their position. Zwicky, in 1937, was the first to achieve the very high probability of identifying a gravitational lens glamour, but until 1979, the first multi-functional imaging device had not yet been found [5]. Since then, dozens of different images have been identified. An instance is shown in Figure 1 [6].

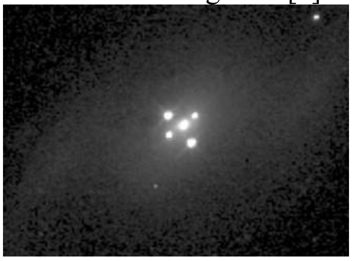

Figure 1. The gravitational lens system 2237+0305 discovered by Huchra et al. shows the situation where an observer sees the lensed images of a distant quasar along the directions of light rays deflected by a massive intervening galaxy [6].

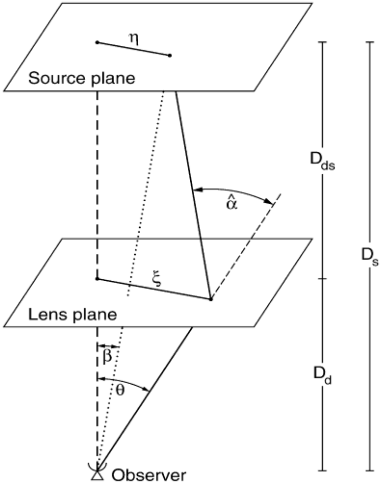

Figure 2. Sketch of a typical gravitational lens system.

Since Einstein's general theory of relativity was proved to be correct, light can pass through "zeros" (null geodesy) on time and space scales. However, most of the situations examined in astrophysics are relatively accessible to light, which is the so-called gravitational lens principle. In Figure 2, a representative case of matter density between redshift $\boldsymbol{z_d}$(or angular diameter distance $\boldsymbol{D_d}$) and beam redshift $\boldsymbol{z_s}$ (or angular diameter distance $\boldsymbol{D_s}$) is considered to be in a typical gravitational lens system. If there are no other refractors around the line of sight, and the variation of the refractive index between the refractive mirror and the light source is much smaller than the angular diameter distance which is defined as $\boldsymbol{D_{ds}}$ and also the angular distance $\boldsymbol{D_d}$, between the refractive mirror and the light source, he real refractive index around the refractor can be replaced by a straight line that produces wrinkles around the two refractors. The magnitude and orientation of the distortion can be expressed by an angle $\boldsymbol{\hat{\alpha}}$, which is related to the particle of the deflection body and the incident vector of the ray.

An equation is necessary to correlate the real location of the center of the source with the location it has observed in the air. As indicated in Figure 2, the plane of the light source and the lens is defined as a plane orthogonal to the optical axis of the light source and the lens. Because there are only a few angles in the general lens case, the accurate optical axis is not needed, and the distance between the lens and the lens is clearly defined in the very fine material distribution [7]. The η is defined to give the 2D position of the source on the source plane.

$$\eta = D_s D_d \xi \ - D_{ds} \hat{\alpha}(\xi) \tag{1}$$

By entering the angular coordinates, $\boldsymbol{\eta = D_s \beta}$ and $\boldsymbol{\xi = D_d \theta}$, it is possible to convert Eq. (1) to

$$\beta = \theta - D_{ds} D_s \hat{\alpha}(D_d \theta\ ) \equiv \theta - \alpha(\theta) \tag{2}$$

The scaled deflection angle $\boldsymbol{\alpha(\theta)}$ will be defined at last. The definition of the lens equation above is the source having an actual position $\boldsymbol{\beta}$ that can be viewed in angular positions $\boldsymbol{\theta}$, satisfying Eq. (2). If there are more than one solution for Eq. (2) at fixed $\boldsymbol{\beta}$, then the sky will show multiple images at several positions for a source at $\boldsymbol{\beta}$. This requires a strong lensing. With the help of dimensionless surface mass density, this can be quantified.

$$\kappa(\theta) = \sum(D_d \theta\ )\,\Sigma_{\mathrm{cr}} \text{ with } \Sigma_{\mathrm{cr}} = \mathrm{c}^2 4\pi \mathrm{G} D_s D_d D_{ds} \tag{3}$$

The redshifts of lens and source have a strong influence on the critical surface mass density which is defined as $\boldsymbol{\Sigma_{cr}}$. A mass distribution where $\boldsymbol{\kappa \geq 1}$ and $\boldsymbol{\Sigma \geq \Sigma_{cr}}$, makes several images for selected source positions $\boldsymbol{\beta}$ [8]. The scaled deflection angle, in terms of κ, can be shown as

$$\alpha(\theta) = 1\pi \int R^2 d^2\theta' \kappa(\theta')\, \theta - \theta' \left|\theta - \theta'\right|^2 \tag{4}$$

Eq. 4 shows that the slope of the deflection potential can be used to record the deviation angle.

$$\psi(\theta) = 1\pi \int R^2 d^2\theta' \kappa(\theta') \ln\left|\theta - \theta'\right| \tag{5}$$

As $\boldsymbol{\alpha = \nabla\psi}$, the potential $\boldsymbol{\psi(\theta)}$ is a 2D simulation of Newtonian gravitational potential, which satisfies the Poisson's equation $\boldsymbol{\nabla^2 \psi(\theta) = 2\kappa(\theta)}$. The deflection angle, $\boldsymbol{\alpha}$, is provided by an integral part of the projected surface mass density, $\boldsymbol{\Sigma(\theta)}$, over angular position,

$$\alpha(\theta) = 1\pi \int d\theta' \frac{|\theta - \theta'|}{|\theta - \theta'|^2} \frac{\Sigma(\theta')}{\Sigma_{cr}} \tag{6}$$

The light composed of multiple images generated by strong lenses travels through different paths, so it takes different times to propagate to us. If the light source is time-varying, such as Quasar and explosion transient, the Pseudo-range multiliterate difference between multiple images could be seen. For each image, the time delay calculation formula is as follows:

$$\Delta t = \frac{1 + Z_l}{c} \frac{D_s D_d}{D_{ds}} \left[\frac{(\theta - \beta)^2}{2} - \psi(\theta)\right] \tag{7}$$

3. Applications

The Development of Gravitational Lenses opens up new applications, including gravitational wave investigation and detection for dark matter, which could lead improved measurements of cosmological parameters.

3.1. Strong lensing of explosive transients for gravitational wave

The From Albert Einstein's theory, he put forward the theory of gravity waves in 1916. Fundamentally speaking, gravitational wave is a kind of curvature fluctuation in space and time, but its stress amplitude is very small, so it is very difficult to detect gravitational wave. The event, known as GW150914, is a pair of two stars in the redshift region of about $\boldsymbol{z \sim 0.09}$. One of the black holes in the two stars fuses between the two stars, with masses of about $\boldsymbol{36\ M_\odot}$ and $\boldsymbol{\sim 29\ M_\odot}$. Figure 3 shows the waveforms detected by GW150914 in the Hanford and Livingston detectors [9].

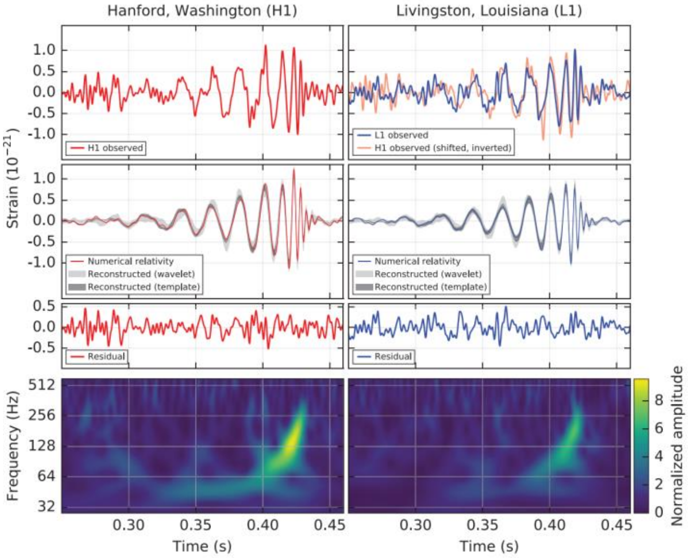

Figure 3. The advanced LIGO detected the first gravitational wave event GW150914. Both Hanford (left) and Livingston (right) detectors detected this event [9].

Therefore, it is possible use gravitational waves to study many properties of merging binaries. The (redshifted) chirp mass, which is considered as one of the most important physical terms, can be strictly limited from observations of gravitational waves.

$$\boldsymbol{M_z = (1+z)M = (1+z)\frac{(m_1m_2)^{3/5}}{(m_1+m_2)^{1/5}}} \tag{8}$$

Here, $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ the masses of two stars made up of two dense matters. In a certain frequency range, the orbit change of pulse emission only depends on the LFM quality of the front end of the pulse, so the quality of pulse transmission is strictly limited. The waveforms of fusion stage and attenuation stage are studied, and, between $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$, it is found that the degeneracy relationship is destroyed. Furthermore, through the research of these waveforms, some information about these spin states can be obtained. More significantly, $\boldsymbol{D_L}$, as the luminosity distance of binary stars, can be determined by gravity waves. This is mainly due to the fact that the occurrence frequency of merging binaries and their changes over time can accurately define the mass of short stars, thus predicting the amplitude of gravitational waves. The amplitude is reduced by the transmission of gravitational waves at the rate $\boldsymbol{{D_L}^{-1}}$ and $\boldsymbol{D_L}$ is limited by the amplitude of gravitational wave. In addition, the redshift cannot be measured directly through the observation of gravitational waves. Therefore, the redshift for GW150914, which is in the redshift region of about $\boldsymbol{z \sim 0.09}$, was actually the ranging data obtained by luminosity distance measurement.

Because a powerful gravitational lens expands the background light source, it allows scientists to observe distant celestial bodies that cannot be seen before. Therefore, under such a high redshift, scientists can limit the speed of redshift of a supernova with strong lensing. The lens looking for high redshift supernovae can be used to observe massive galaxy clusters. The above results show that the magnification factor of the gravitational lens has a certain influence on the observed explosive transients. This is particularly critical for gravity waves generated by the fusion of two black holes, and in most cases, the redshift cannot be directly observed. On the contrary, the observation of gravitational waves can be used to measure the luminosity distance to the source. When the magnification of the gravitational lens is $\boldsymbol{\mu}$, the observed luminosity distance is corrected to

$$\boldsymbol{D_L}^{\boldsymbol{obs}} = \frac{D_L}{\sqrt{\mu}} \tag{9}$$

Here, $\boldsymbol{D_L}$ is the luminosity distance at the source when there is no gravitational lensing effect (that is, under the assumption of consistent isotropic cosmology, the luminosity distance measured by gravitational waves) and $\boldsymbol{D_L}^{\boldsymbol{obs}}$ is the luminosity distance measured from observations of gravitational waves. Therefore, the redshift inferred from the luminosity distance, for highly magnified events $\boldsymbol{\mu} \gg \mathbf{1}$, is biased low. According to the chirp mass estimated by Eq. (8), the bias in estimating redshift will have a direct impact on it.

3.2. Dark matter

The results of cosmology show that dark matter (DM) makes up about 84% of the universe, and its gravitational effect can explain the existence of dark matter. However, there is still a lack of effective description of it in the mainstream particle physics theory. The nature of DM and the interaction between them has always been the most puzzling problem in the field of basic physics. Strong gravitational lensing is a potential observation tool, which can measure dark matter fluctuations of different scales, especially in general dwarf galaxies.

When the gravitational lensing is used for observation, there is a certain relationship between the radiation quality of the perturbation and the observed value, which is due to the relationship between the radiation quality of the perturbed body and the observed value. The distance from the perturbator(perturber) to the critical curve in the direction of maximum disturbance to amplification is defined as the perturbation radius. The effective quality of the perturbed lens (that is, the projected quality within the perturbed radius, divided by the logarithmic slope of the 2D density distribution of the main lens) has been proved to be equivalent to a robust estimator of the quality of the perturber when the density distribution changes. The information above helps scientists to measure perturber masses.

While perturber mass is measured, its density can also be gotten through its inversely relationship with radius. In particular, the average convergence gradient of perturbers is very sensitively influenced by perturbers in strong lens systems at the feature scale so scientists hope to see that the disturbance has the greatest impact on the observation. This is called as the effective density slope [10]. The slope of this effect is expected to be different for different DM scenes, so the measurement of the slope of this effect can lay a solid foundation for distinguishing different DM models.

Figure 4 and Figure 5 show a strong lens system JVAS B1938+666 which was detailed investigated [11]. The effective slope of the density profile of the perturber in the strong lens system JVAS B1938+666 was measured,

$$\kappa(x,y) = \frac{3-\gamma}{2}\left(\frac{\theta_E}{\sqrt{qx^2+y^2/q}}\right)^{\gamma-1} \tag{10}$$

where $\boldsymbol{\gamma}$ is the negative power-law slope of the 3D mass distribution, $\boldsymbol{\theta_E}$ is the Einstein radius, $\boldsymbol{x}$ and $\boldsymbol{y}$ are the angular coordinates aligned with lens principal axis and the secondary principal axis, and $\boldsymbol{q}$ is the ratio of the principal axis to the secondary principal axis. Because ETHOS1-3 is subjected to stress to a large extent, its slope also exceeds that expected by the model. It is also $\boldsymbol{a} \approx \boldsymbol{3\sigma}$ outlier for ETHOS4 and $\boldsymbol{a} \approx \boldsymbol{2\sigma}$ outlier for CDM. The tension of concentration is equivalent to this tension. The greater the slope is, the greater the density is, and vice versa in NFW halo. Because the perturber with larger effect gradient (larger density) has stronger gravitational lensing signal, it is easier to be observed.

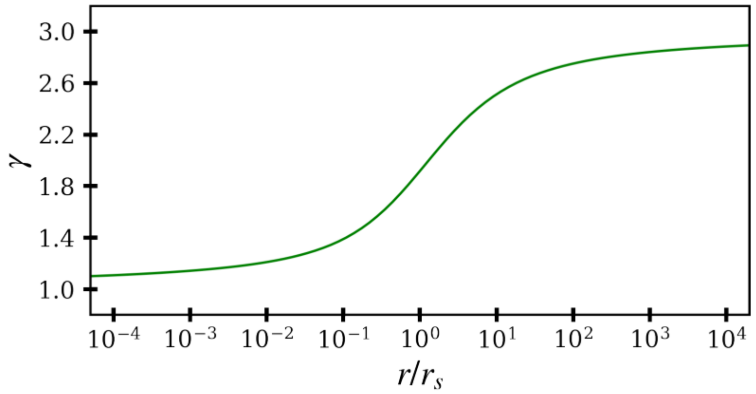

Figure 4. The effective slope of the NFW profile is a function of the radius represented by the scale radius r_s [11].

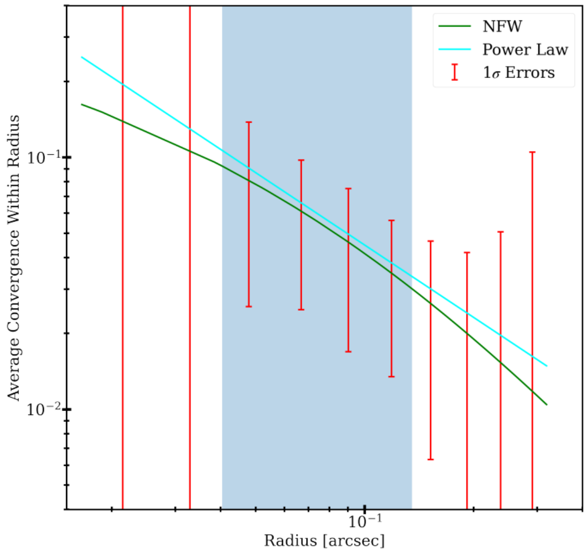

Figure 5. The linear errors for the perturber of the system JVAS B1938+666. The power-law best fit with $\gamma = 1.95$ is shown in cyan line. The NFW best fit is shown with the green line [11].

4. Limitations & prospects

From time delays precise measurements of H_0 could be gotten by The precise measurements of image positions which can definitely be improved by those strong lensing of explosive transients. However, in some cases, accurate measurement of image position can be difficult. This is because the angular resolution of the observation results to detect those explosive transients is poor. One way to get accurate astronomical observation data is to find out the corresponding other bands, especially the corresponding bands. The optical equivalent can be applied to gravitational waves produced by the merging of two dense matters, such as neutron star, and the accurate location of millimetre-angle-second images can be realized through high-precision deep space detection of multiple images, in order to achieve the accurate detection of time delay cosmology.

Although the problems existing in the high-intensity gravitational field at the moment of the great explosion can be explained in theory, it is still difficult to find such stars in practical application. Even though a lot of research work has been carried out in the world, there is still no reliable candidate source for a variety of gamma ray sources. The search in 2019 used the sample gamma-ray burst, which is also an example of explosive transients, detected in 1995 [12, 13], which has a high working cycle and can search for lens image matching in a wide range. In addition, the microscopic lens will cause the distortion of the light curve, which leads to a large deviation in the similarity retrieval of multiple image pairs using the light curve. Although the possible cases of strong gravitational wave fast radio storms have been estimated in reference [14], it has not been systematically studied so far. As more and more high-speed radio storms are observed, the search for real stars will be interesting in the future.

In experiments, the usual method is to assume the expansion profile of galaxies, so as to get the observations of galaxies. In some cases, due to the influence of various assumptions, the calculated matter will have an order of magnitude change, thus affecting the accurate measurement of dark matter (subhalo). In addition, the disturbance caused by the gravitational wave generated by the gravitational wave in the gravitational wave is called "effective density tilt" for the convergence of the gravitational wave. However, this measurement method still needs to be verified. More of new strong lensing systems with experiments are expected to be found. Due to the increasing ability of observing strong lensing systems, it is expected to discover more perturbers. This project will use the observation data of HST, James Weber Space and Extreme Large Telescope, combined with the observation data, to give the actual observation data, and compare with the numerical calculation results in order to achieve appropriate results.

5. Conclusion

To sum up, gravitational lensing provides opportunities for various fields of astronomical further research. In this article, the development history of Gravitational lensing is briefly reviewed, and its basic principles, including lens equation, deflection angle and time delay, are simply introduced. This article then focuses on its two modern advanced applications: strong lensing of exploratory transitions for gravitational waves to investigate red shift, and dark matter exploration by measuring effective density slope of the perturber. In addition to Gravitational wave, other explosive transients include Gravitational lens, which also have great potential, but there is not enough exploration for these lenses at present. For dark matter detection, the shortcomings of conventional methods are difficult to avoid, and new measurement methods require more sets of data and experiments to support them. It is vital to find more Gravitational lens and carry out more measurements and experiments on them. It will deepen the modern view of the Universe in several ways in coming years. With more detection methods, it is able to gain a deeper understanding of the characteristics of gravitational lenses and thus better understand the universe. Through the analysis of dark matter and explosive transients, new ideas to understand the unknown phenomena in the universe will be introduce

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