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Research and analysis of problem-solving strategies of conic section in high school mathematics

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Abstract. The conic section is a crucial part of high school mathematics and usually plays the role of tough problem in examinations. Hence, this dissertation will use the literature analysis method to introduce the mathematical background of the conic section to enable students to know about the conic section more clearly and explain the basic concept of the conic section such as the first and second definitions of conic section, then introduce some second level conclusions such as focus length formula and average property of parabola. After that, this dissertation will provide some conic section problems in past exam papers and corresponding problem-solving processes to show how to use the basic concept and these common second-level conclusions of the conic section, then summarize the train of thought of the conic section. For choice questions and fill-in-the-blank questions, combining the condition given by questions with the first definition and second common level is important, and for questions that need to write the complete process, setting parameters and Vieta theorem is essential, and for extremely complicated problems, special methods such as homogenization can be considered to reduce the calculation.

Keywords: Conic Section, Eccentricity, Vieta Theorem.

1. Introduction

The mathematical background of the conic section is extremely rich. The research on the conic section can be traced back to Ancient Greece [1]. The cubic problem which is one of the three major geometric problems was considered closely related to conic section by people at that time, and the cubic problem was transformed by foreign scholar Hippocrates into the quadratic ratio problem, and then Menaechmus analyzed the quadratic ratio problem based on the curve interaction [1]. This curve is called the conic curve [1]. Subsequently, Archimedes and others continued to research the content of conic sections and wrote a famous book called *Treatise on Conic Sections* which systematically explains the concept of conic section [1]. For more than 1,000 years since then, the research on the conic section developed sluggishly, and research results about the conic section were closely related to physics [1]. For instance, in the 16th century, Galileo proposed that the trajectory of an object doing oblique throwing motion is a parabola [1]. Moreover, Kepler proposed that the trajectory of astronomical objects is elliptic [1]. In the 17th century, the advent and development of analytic geometry enabled the algebraic methods and conic section to be increasingly related [1]. Euler analyzed the equation $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ and the types of conic sections were divided by Euler based on a new perspective [1].

The conic section is a crucial part of high school mathematics, and always plays the role of a tough problem in college entrance examination mathematics [2]. In junior high school, the conic section has appeared, which is a quadratic function [1]. After the new curriculum reform, the main content of analytic geometry is a conic section which is based on the content of straight lines and circles [1]. Later, the basic course of a college mathematics major also includes the conic section, which effectively connects elementary mathematics with higher mathematics, so in middle school mathematics, the conic section plays a pivotal role, especially in analytic geometry of high school mathematics [3].

This dissertation will start with definitions, standard equations, geometric properties, and some conclusions based on basic knowledge of the conic section to give the strategy of solving different problems of the conic section.

This dissertation will research with the literature analysis method and complete the following tasks:

Know basic knowledge of the conic section.

Summarize the common question types in the Nationwide Unified Examination for Admissions to General Universities and Colleges and simulation examination papers.

Give different problem-solving strategies for different problems of the conic section.

The significance of this research is that it can help high-school students to know how to realize the question type of conic section they meet when doing test paper, so they can know what geometric properties and conclusions of the conic section should be used, and hence they are able to solve the conic section problem in less time and have more time to solve other tough problems.

The research objective of this dissertation is to summarize the common question types of conic section and find the common strategy to solve these questions based on definition, geometric properties, and some conclusions of conic section.

2. Research and analysis of problem-solving strategies of conic section

2.1. Concepts involved in the conic section of high school mathematics

2.1.1. Basic concepts of conic section. The conic section is a curve obtained from a plane truncated quadric conical surface and includes an ellipse, hyperbola, and parabola. The standard equation of the ellipse is equation 1:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b>0) \tag{1}$$

if foci are on the x-axis and $\frac{y^2}{a^2}+\frac{x^2}{b^2}=1$ (a>b>0) if foci are on the y-axis.

Wherein, a represents the length of the semi-major axis and b represents the length of the semi-minor axis. The standard equation of the hyperbola is:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\ (a>0,b>0)$$

if foci are on the x-axis and $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ (a>0, b>0) if foci are on the y-axis.

A represents the length of the semi-real axis and b represents the length of the semi-imaginary axis. Furthermore, hyperbola has two asymptotes y=±(b/a)x if foci are on the x-axis and y= ±(a/b)x if foci are on the y-axis. The standard equation of a parabola is slightly different as it has four forms and only one focus. For the right and left opening parabolas. standard equation is equation 2:

$$\mathrm{y}^2=2\mathrm{px}\text{ and }\mathrm{y}^2=-2\mathrm{px}\ (\mathrm{p}>0) \tag{2}$$

For upper and lower parabolas, standard equations are

$$\mathrm{x}^2=2\mathrm{p}y\text{ and }x^2=-2\mathrm{p}y\ (\mathrm{p}>0)$$

p represents focal distance.

Except for these standard equations, the conic section has some crucial concepts used frequently in conic section problems. All three different conic sections have first definitions. For an ellipse, the first definition is the trajectory of the moving point the sum of the distance from this point to two fixed points F1 and F2 is a constant, and F1 and F2 are two foci of ellipse [4]. Its mathematical expression is: |PF1|+|PF2|=2a. For hyperbola, it is the trajectory of the moving point that the difference of distance from this point to two fixed points F1 and F2 is a constant, and F1 and F2 are two foci of hyperbola. Its mathematical expression is: ||PF1|-|PF2||=2a. For a parabola, it is the trajectory of the moving point that the distance from this point to one fixed point and one fixed line is equivalent (this fixed line does not pass through the fixed point).

Conic sections also have their second definitions which are that the ratio of distance from the point on curves to the focus and the fixed line is a constant [5]. This ratio is called eccentricity, written as e and equal to $\frac{c}{a}$ (In an ellipse, $b^2 = a^2 - c^2$, and in hyperbola, $b^2 = c^2 - a^2$). The eccentricity of ellipse is greater than 0 and less than 1, that of parabola and hyperbola are equal to one and larger than one. This fixed line is called directrix. In ellipse and hyperbola, the equation of directrix is $x = \pm\frac{a^2}{c}$ when foci are on the x-axis and $y = \pm\frac{a^2}{c}$ when foci are on the axis. The equation of directrix of parabola is $x = -\frac{p}{2}$ or $x = \frac{p}{2}$ when the parabola is right opening or left opening and $y = -\frac{p}{2}$ or $y = \frac{p}{2}$ when the parabola is upper opening or lower opening.

2.1.2. The common second-level conclusion of the conic section. The second level conclusion can be deduced from the basic concept of the conic section, which is extremely convenient for solving conic section problems. Due to the fact that the main objective of this dissertation is to give a problem-solving strategy for common problems, so will not give every proof of the conclusions below. One common conclusion is the third definition of ellipse and hyperbola which is the trajectory of the point that the product of gradients of this point to two fixed points such as two points that ellipse and hyperbola intersect with semi-major axis is a constant [6]. For an ellipse, this constant is $-\frac{b^2}{a^2}$ when foci are on the x-axis and $-\frac{a^2}{b^2}$ when foci are on the y-axis. For an ellipse, this constant is $\frac{b^2}{a^2}$when foci are on the x-axis and $\frac{a^2}{b^2}$ when foci are on the y-axis. Actually, this third definition can be extended. The product of gradients of the point on the ellipse or hyperbola and two points that the line passing through the origin intersects with the ellipse or hyperbola, and the product of gradient of the chord of ellipse or hyperbola and the line passing through the origin and midpoint of this chord are also constants equal to the values above. The ellipse and hyperbola also have another very common conclusion which is the area of focus triangle. The focus triangle is composed of one point P on the curve and two foci F1 and F2. The area of focus triangle in the ellipse is

$$b^2 \tan\left(\frac{\theta}{2}\right), \theta = \angle F1PF2.$$

Assume PF1=m and PF2=n, in ellipse, PF1+PF2=m+n=2a, F1F2=2c.

From the law of cosines, $cos\theta = \frac{m^2+n^2-4c^2}{2mn}$, so $cos\theta = \frac{[(m+n)^2-2mn-4c^2]}{2mn}$ and $2mncos\theta = 4a^2 - 2mn - 4c^2$, $2(cos\theta + 1)mn = 4b^2, mn = \frac{2b^2}{cos\theta+1}$

The area of triangle.

$$PF1F2 = \frac{1}{2mnsin\theta} = \frac{1}{2sin\theta} \times \frac{2b^2}{cos\theta+1} = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) \times \frac{b^2}{2cos^2\left(\frac{\theta}{2}\right)} = \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} \times b^2 = b^2\tan\left(\frac{\theta}{2}\right).$$

The area of focus triangle in hyperbola is $b^2\cot\left(\frac{\theta}{2}\right)$ [7].

The tangent equation of the conic section is also very crucial. Generally, assume the point of tangency is (x_0, y_0), the tangent equation of quadratic curve equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is

$Ax0x + B\frac{x0y+xy0}{2} + Cy0y + D\frac{(x+x0)}{2} + E\frac{(y+y0)}{2} + F = 0$. Most conclusions above are only suitable for ellipse and hyperbola. However, parabola also has some conclusions and properties that ellipse and hyperbola do not have. The form of conclusions depends on the opening of the parabola, so this dissertation only takes the right opening parabola as an example, the length of the chord of the right opening parabola is $\frac{2p}{\sin^2(\theta)}$, θ is the angle between the chord and x-axis. Based on this, the area of the triangle is composed of two endpoints and the origin is $\frac{p^2}{2sin\theta}$. Another conclusion of the right opening parabola is $y0 \times kAB = p$, A and B are two endpoints of the chord of the parabola and y0 is the ordinate of the midpoint of this chord. Assume the chord of the right and left opening parabola has two endpoints A (x1, y1) and B (x2, y2), and intersect the x-axis with point T (m,0), then $x1x2 = m^2$and $y1y2 = -2pm$. This is the average property of a parabola.

2.2. Common conic section problems in examinations and corresponding problem-solving strategies

Usually, the strategy of solving choice questions and fill-in-the-blank questions is different from that of questions that need to write whole complete problem-solving process. Hence, this dissertation will first give the strategy to solve choice questions and fill-in-the-blank questions.

This dissertation will give some problems of different common types of conic section problems and the corresponding problem-solving process, then summarize the train of thought of these problems. Some conic section problems only involve the definition of conic section. For these problems, knowing about the first definition of conic section is essential. One problem is: Given that F1 and F2 are two foci of ellipse C: $\frac{x^2}{9} + \frac{y^2}{4} = 1$, point M is on this ellipse, find the maximum value of |MF1|×|MF2|. The process is: $|MF1| \times |MF2| \leq \frac{(MF1+MF2)^2}{4} = \frac{4a^2}{4} = 9$, so maximum value is 9. Another problem is: given that F1 and F2 are two foci of hyperbola C, P is on the C, ∠F1PF2=60°, |PF1|=3|PF2|, find the eccentricity of C.

The process is: Assume PF2=m, so PF1=3m. 3m-m=2a, then m=a, $cos\angle F1PF2 = \frac{PF1^2+PF2^2-F1F2^2}{2PF1F2} = \frac{9m^2+m^2-4c^2}{2\times 3m\times m} = \frac{10a^2-4c^2}{6a^2} = cos60° = \frac{1}{2}$ e, $10a^2 - 4c^2 = 3a^2, 7a^2 = 4c^2, \frac{c^2}{a^2} = \frac{7}{4}, e = \frac{c}{a} = \frac{\sqrt{7}}{2}$.

This type of conic section problem is an easy-scoring question as it only involves the first definition of the conic section without any other complicated concepts. For this type of problem, using manners such as basic inequality and the law of cosines to transform the question into the first definition of the conic section is extremely helpful.

Then, this dissertation will give some conic problems related to second-level conclusion and complicated calculations. These problems are often the final problem in choice questions and fill-in-the-blank questions.

Problem: Given that the left vertex of ellipse C: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a > b > 0)$ is A, points P and Q are on C and symmetrical about y-axis, the product of the gradient of AP and AQ is equal to 1/4, find the eccentricity of C. Process: Find a new point M, and M, Q are symmetrical about x-axis, so gradient of AP×gradient of AM=-gradient of AP×gradient of $AQ = -\frac{1}{4} = -\frac{b^2}{a^2}$, so $\frac{a^2-c^2}{a^2} = \frac{1}{4}, 1 - e^2 = \frac{1}{4}, 4 - 4e^2 = 1, e^2 = \frac{3}{4}, e = \frac{\sqrt{3}}{2}$. Problem: Given that points F1, and F2 are two foci of hyperbola: $x^2 - \frac{y^2}{3} = 1$, P is on the right part of hyperbola and tan∠F1PF2=4√3, O is the origin, find the length of OP. Process: Assume ∠F1PF2=θ, $tan\theta = 2\tan(\theta/2)/(1 - \tan\ \hat{}2(\theta/2))$,

So $4\sqrt{3} - 4\sqrt{3}\tan\ \hat{}2(\theta/2) = 2\tan(\theta/2)$, $2\sqrt{3\tan\left(\frac{\theta}{2}\right) + \tan\left(\frac{\theta}{2}\right) - 2\sqrt{3}} = 0$, $\Delta = b^2 - 4ac = 1 + 48 = 49$. Due to the fact that $\frac{\theta}{2}$ is an acute angle, so $\tan\left(\frac{\theta}{2}\right) > 0$, tan(θ/2)=$\frac{\sqrt{3}}{2}$, the area of triangle

F1PF2= $\frac{b^{2cot}}{\frac{\theta}{2}} = 2\sqrt{3} = \frac{1}{2} \times |y(p)| \times 2c = c \times |y(p)| = 2|y(p)|$ o $|y(p)| = \sqrt{3}, x^{2(p)} = 2$. Hence, OP=$\sqrt{2+3} = \sqrt{5}$.

Problem: given that ellipse C: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a > b > 0)$, the upper vertex of C is A, two foci are F1 and F2, eccentricity is 1/2, the line passing through point F1 and perpendicular to AF2 intersects C with two points D and E, |DE|=6, find the circumference of triangle ADE. Before giving the process of this problem, this dissertation will introduce the focus chord formula in ellipse: the length of the focus chord of ellipse that foci are in x-axis=$\frac{2ab^2}{a^2 - c^2 \cos^2\theta}$, θ is the angle between the focus chord and x-axis. Based on this formula, this problem can be solved quickly.

Process: $e = \frac{c}{a} = \frac{1}{2}$, $a = 2c$, $b^2 = a^2 - c^2 = 3c^2$, so $b = \sqrt{3c}$. Hence, the coordinate of A is (0, √3c), F1A=F1F2=2c, so F1 is on the perpendicular bisector of AF2. Due to the fact that line DE is perpendicular to AF2 and passed through F1, so line DE is the perpendicular bisector of AF2, DA=DF2, EA=EF2.

Hence, the circumference of triangle $ADE = AD + DE + AE = DF2 + EF1 + DF1 + EF2 = (DF2 + DF1) + (EF1 + EF2) = 4a$.

Let O be the origin of the coordinate system, so OA=√3c, OF2=c, so ∠AF2O=60°, ∠DF1F2=30°.

Hence, |DE|=$\frac{2ab^2}{a^2 - c^2\ \cos^2 30°} = 6$, $\frac{2ab^2}{a^2 - \frac{3c^2}{4}} = 6$, $\frac{2\times 2c\times 3c^2}{4c^2 - \frac{3c^2}{4}} = 6$, $12c^3 = \frac{39c^2}{2}$, $c = \frac{13}{8}$, $a = 2c = \frac{13}{4}$.

Hence, the circumference of the triangle ADE=4a=13. Problem: for parabola E: y^2=4x, there are two different lines L1 and L2 passing through focus F, L1 intersects E with points A and C, and L2 intersects E with points B and D, assume line AB and line CD intersect M (x1,0) and N (x2,0) respectively, find the value of x1·x2.

Process: $2p = 4, p = 2, \frac{p}{2} = 1$, so the coordinate of focus F is (1,0), $x(A) \cdot x(C) = x^2(F) = 1$, $x(B) \cdot x(D) = x^2(F) = 1$, $x(A) \cdot x(B) = x^2(M) = (x1)^2, x(C) \cdot x(D) = x^2(N) = (x2)^2$, so $x(A) \cdot x(B) \cdot x(C) \cdot x(D) = (x1)^2 \cdot (x2)^2 = 1, x1 \cdot x2 = 1$.

Through these problems, the common type of choice questions and fill-in-the-blank questions of the conic section and corresponding problem-solving can be summarized. Basic questions, usually only involve the basic concept of conic section, the method to solve these questions is to utilize a formula such as the law of cosines to establish the relationship between each line segment and use the first definition to find the value needed to be found, such as eccentricity. For tough questions, they usually involve complex geometric relationships and complex calculations if only using the basic concept of the conic section. Utilizing the condition in questions and corresponding second-level conclusions to build the relationship between each condition, then based on some other relationships built by other formulas, the problem can be solved more quickly. For instance, if the question gives the length of the focus chord and asks to find the value of eccentricity, the focus chord formula should be considered, then try to find some special relationship such as perpendicular bisector based on conditions in question. Hence, the equation about a, b, and c will be found, then based on $b^2 = a^2 - c^2$ in ellipse and $b^2 = c^2 - a^2$ in hyperbola, the value of eccentricity will be found.

The conic section problem that needs to write a complete problem-solving process is much more complicated than choice questions and fill-in-the-blank questions as this problem has various types and has abnormally large calculations if using common methods to solve. Hence, the dissertation will only focus on a few types of problems. After giving the common problem-solving strategy, this dissertation gives some special methods which can solve some conic section problems quickly.

Usually, the strategy of solving a section problem that needs to write the whole process is: setting the parameter such as the coordinate of moving point and equation of moving line or curve, then setting the simultaneous equation and using the Vieta theorem to build a relationship between the parameter, then based on the condition in question, the problem can be solved. However, sometimes the conic section problem is extremely complicated, and very difficult to build suitable relationships between parameters, so this dissertation will only give the examples of relatively simple questions and questions that can be solved by special methods directly.

Problem: Given that the eccentricity of ellipse C: $\frac{y^2}{a^2}+\frac{x^2}{b^2}=1(a>b>0)$ is √5/3, point A (-2,0) is on C. (1) find the equation of C. (2) the line passing through point (-2,3) intersect C with two points P and Q, line AP and AQ intersect y-axis with M and N respectively, prove that the midpoint of MN is fixed point.

Process: (1) $e=\frac{c}{a}=\frac{\sqrt{5}}{3},\frac{c^2}{a^2}=\frac{5}{9},c^2=\frac{5a^2}{9}$. A (-2,0) is on C, so $\frac{4}{b^2}=1$, $b^2=4=a^2-c^2=\frac{4a^2}{9}$, $a^2=9$. Hence, the equation of C is $\frac{y^2}{9}+\frac{x^2}{4}=1$. (2) Let the line be $y=k(x+2)+3$, setting the simultaneous equation between $y=k(x+2)+3$ and $\frac{y^2}{9}+\frac{x^2}{4}=1$.

So $4(kx+2k+3)^2+9x^2=36, m(4k^2+9)x^2+(16k^2+24k)x+(16k^2+48k)=0$.

Hence, from Vieta theorem, $x1+x2=-\frac{16k^2+24k}{4k^2+9}$, $x1\cdot x2=\frac{16k^2+48k}{4k^2+9}$. The equation of AP is

$\frac{x+2}{x1+2}=\frac{y-0}{y1-0}$,

so $y=\frac{y1(x+2)}{x1+2}$,

when $x=0$, $y=\frac{2y1}{x1+2}$,

so the coordinates of M is $\left(0,\frac{2y1}{x1+2}\right)$, N is $\left(0,\frac{2y2}{x2+2}\right)$,

the ordinate of midpoint of MN is

$$\frac{\frac{2y1}{x1+2}+\frac{2y2}{x2+2}}{2}$$
$$=\frac{y1x2+x1y2+2(y1+y2)}{x1x2+2(x1+x2)+4}$$
$$=\frac{\left[\frac{-32k^3-48k^2-72k}{4k^2+9}+8k+12\right]}{\left[\frac{36}{(4k^2+9]}\right]}$$
$$=\frac{\frac{-32k^3-48k^2-72k+32k^3+72k+48k^2+108}{4k^2+9}}{\frac{36}{4k^2+9}}$$
$$=\frac{\frac{108}{4k^2+9}}{\frac{36}{4k^2+9}}=\frac{108}{36}$$
$$=3.$$

Hence, the midpoint of MN is fixed point (0,3). It can be seen that the calculation is abnormally large if using conventional problem-solving methods, so this dissertation will provide two special ways of solving problems, which can tremendously reduce the calculation.

The first method is homogenization. That is, translating the coordinate system to make the fixed point become the origin. Problem: Given that A (2,1) is on hyperbola C: $\frac{x^2}{a^2}-\frac{y^2}{a^2-1}=1\ (a>1)$, line L intersects C with point P and Q, the sum of the gradient of AP and AQ is 0. Find the gradient of L.

Process:

$\frac{4}{a^2}-\frac{1}{a^2-1}=1$,

$4(a^2-1)-a^2=a^4-a^2$,

$3a^2-4=a^4-a^2, a^4-4a^2+4=0$,

$(a^2-2)^2=0, a^2=2, a=\sqrt{2}$.

So the equation of C is $\frac{x^2}{2}-y^2=1$. Translating all the point in the coordinate two units to the left and one unit down, so the new coordinate of A will be (0,0), written as A', Assume the equation of line P'Q' (line L') be $mx+ny=1$, P' $(x1,y1)$, Q' $(x2,y2)$, the equation of C' is $\frac{(x+2)^2}{2}-(y+1)^2=1$, then setting the simultaneous equation of C and line P'Q', for C, $\frac{x^2+4x+4}{2}-y^2-2y-1=1$,

$x^2+4x+4-2y^2-4y-2-2=0$,

$x^2+4x-2y^2-4y=0$,

$x^2-2y^2+(4x-4y)=0, mx+ny=1$, so it can be written as $x^2-2y^2+(4x-4y)\cdot(mx+ny)=0$, $x^2-2y^2+4(mx^2-mxy+nxy-ny^2)=0$, $x^2-2y^2+4mx^2-4mxy+4nxy-4ny^2=0$, $(1+4m)x^2-(2+4n)y^2+(4n-4m)xy=0$, $(2+4n)y^2-(4n-4m)xy-(1+4m)x^2=0$, $(2+4n)\cdot\left(\frac{y}{x}\right)^2-(4n-4m)\cdot\left(\frac{y}{x}\right)-(1+4m)=0$, so $\frac{y1}{x1}+\frac{y2}{x2}=\frac{4n-4m}{2+4n}$. Due to the fact the sum of gradient of A'P' and A'Q' is 0, $\frac{y1}{x1}+\frac{y2}{x2}=\frac{4n-4m}{2+4n}=0$, 4n-4m=0, n=m. Hence, for L': $mx+ny=1, ny=1-mx, ny=1-nx, y=\frac{1}{n}-x$, the gradient of L'is -1, so the gradient of L is also -1. When the question involves one fixed point and the sum or product of the gradient of two lines passing through the fixed point and moving points, which is a constant, homogenization can be considered as it can reduce the calculation [8]. The train of thought of this method is: translate the coordinate system to enable the fixed point to be the origin, then assume the equation of moving line be $mx+ny=1$, then let it multiply with the non-quadratic terms in equation of conic section to make all terms in equation of conic section are quadratic, then divided by x^2, this equation will become a quadratic equation about y/x, and use Vieta theorem to build the relationship of sum or product of $\frac{y1}{x1}$ and $\frac{y2}{x2}$. This represents the sum or product of the gradient of the fixed point after translating (origin) and two moving points, then based on the condition given by the question, this conic section problem will be solved. From the problem above, it can be seen that if using conventional manners, this sum of the gradient of two lines is $\frac{y1-1}{x1-2}+\frac{y2-1}{x2-2}$ which the calculation is very large, but after homogenization, it will become $\frac{y1}{x1}+\frac{y2}{x2}$, then substitute the value given by the question directly as the translation does not change the gradient of lines. However, sometimes the question may ask to find the coordinate of the point rather than the gradient of the line, so after finding the coordinate of the translated point, this point needs to be translated to the original coordinate system.

Another useful method is pole and polar. Make two secants of conic section that passes through a point P outside the conic section (assume one line passing through P intersects the conic section with points A and B and another line passing through P intersects the conic section with C and D), then join AC and BD to let them intersect with point F and join AD and BC with E, then point P, E and F are called pole and the line passing through any two of these three points are called polar [9]. The form of equation of polar of quadratic equation $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ is the same as tangent equation, which is $Ax0x+B\frac{x0y+xy0}{2}+Cy0y+D\frac{x+x0}{2}+E\frac{y+y0}{2}+F=0$ based on the assumption that the coordinate of one pole is (x0, y0) [10]. Then this dissertation will utilize one problem to demonstrate how to use pole and polar. Problem: Given that points A and B are left and right vertices of ellipse E: $\frac{x^2}{a^2}+y^2=1\ (a>0)$, G is the upper vertex of E and vector AG·vector GB=8, P is a moving point of line x=6, PA intersects E with point C, PB intersects E with point D. (1) find the equation of E. (2) prove: line CD passes through fixed point.

A(-a,0), G (0,1), B(a,0), vector AG=a**i**+**j,** vector GB=a**i**-**j,** vector AG·vector GB=$a^2-1=8$. $a^2=9$, $a=3$. Hence, the equation of C is $\frac{x^2}{9}+y^2=1$. (2) Assume the coordinate of P is (6,0), so the equation of polar about pole P is $\frac{6x}{9}+y0y=1$, when $y=0$, $6x=9$, $x=\frac{3}{2}$, so this polar intersect x-axis with point $\left(\frac{3}{2},0\right)$, the intersecting point of AB and CD is on x-axis, which is also a pole, so the coordinate of this pole is $\left(\frac{3}{2},0\right)$. Hence, line CD passed through a fixed point $\left(\frac{3}{2},0\right)$. It can be seen that the process is much briefer than previous ones. Usually, when the question is about the proof that a line passes through a fixed point and there are two secants of conic section, pole and polar can be considered.

3. Conclusion

The strategy of problem-solving of choice questions and fill-in-the-blank questions is relatively fixed. Generally, these two questions involve the definition of a conic section and some geometric figures. The general problem-solving strategy is to combine the condition given by questions with the first definition of the conic section, and use some formulas existing in geometric figures such as the law of cosines to find the relationship between a, b, and c, then based on $b^2 = a^2 - c^2$ in ellipse and $b^2 = c^2 - a^2$ in hyperbola, the value of a, b and c will be found, then it is very easy to find the value needed to be found. For some complicated questions, second-level conclusions should be considered and second-level conclusion should be used based on the condition given by questions to reduce calculation. For instance, if the question gives the length of the focus chord, the focus chord formula should be considered.

For a conic section problem that needs to write a complete process, the basic train of thought is to set the parameters such as coordinates of moving points and equations of moving lines or curves, then set simultaneous equations of moving lines and curves, and use Vieta theorem to find the relationship between parameters, then solve the problem based on conditions given by questions. Nevertheless, the calculation is abnormally large and sometimes it is extremely tough to find a suitable relationship between parameters to solve the problem. Hence, some special methods can be considered. When the question involves one fixed point and the sum or product of gradient of two lines passing through the fixed point and moving points, which is a fixed value, homogenization is likely to be used, which is translating the coordinate system to enable the fixed point to be the origin, then assume that the equation of line passing through two moving points is $mx + ny = 1$, then multiply this equation with the non-quadratic term in conic section equation to build a quadratic equation about y/x which is the gradient of line passing through the fixed point and one moving point, then the expression of sum or product of gradient will be found based on Vieta theorem. For the proof that a line passes through a fixed point, pole and polar can be considered, which can greatly reduce the calculation.

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