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# An Overview of the Flatness Problem and Its Proposed Solutions

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**Abstract:** As a drawback to the widely accepted standard Big Bang model, the Flatness problem has been highly controversial. Due to the presence of dark matter and difficulty in measuring distances between galaxies, the universe's density in the present day has high uncertainty. There are three models consistent with the cosmological principle. The Flatness problem originated from the uncertainty of the density parameter and the puzzling fine-tuning of omega and the point of the Big Bang; its potential to identify a shortcoming in the standard cosmological model has drawn considerable attention from the scientific community. Different approaches were taken as attempts to resolve this problem. However, arguments against the Flatness problem have also been proposed, disproving the existence of the Flatness problem from the perspective of classical cosmology. This paper examines the Flatness problem by quantitatively describing the flatness problem, presenting the historical development of the Flatness problem, analyzing past approaches to the Flatness problem, and considering arguments against the problem from the perspective of classical cosmology.

**Keywords:** Flatness problem, Big Bang model, cosmology theory.

## 1. Introduction

Since its formation by Dicke in 1970, the Flatness problem has puzzled cosmologists because it suggests a drawback to the widely accepted standard Big Bang models. The standard Big Bang models lie in the framework of Einstein's theory of general relativity that gives the derivation of the Robertson Walker metric and the Friedmann equations describing the cosmic dynamics. [1] Moreover, the standard is based on the cosmological principle, which describes the universe as isotropic and homogenous on a large scale (scale of clusters of galaxies); the cosmological principle essentially states that the universe is identical in every direction and in every point, which further asserts that distribution of matter is approximately the same on a large scale.[2] The Friedmann equation is derived from Einstein's equations, and they are listed as follows:

$$\dot{R}^2 = \frac{8\pi G\rho R^2}{3} + \frac{\Lambda R^2}{3} - kc^2 \tag{1}$$

Where R is the scale factor of the universe and a function of time, G is the gravitational constant, $\rho$ is the universe's density, $\Lambda$ is the cosmological constant, and c is the speed of light. [3] However, the value of the curvature constant k can take on three different values depending on the curvature of space. Possible values of k could be equal to –1, 0, or 1. For further discussions, it is necessary to introduce the

Friedmann equation under matter-dominated epochs, a period when the universe was approximately 380,000 years old when matter dominated the universe.

$$\frac{k}{R_o^2}=\frac{8\pi G\rho_o}{3c^2}-\frac{H_o^2}{c^2} \tag{2}$$

Note that $R_o$ and $\rho_o$ refer to the value of these variables in the present day. H is defined as $H(t)=\frac{\dot{R}(t)}{R(t)}$, and $H_o$ is equivalent to $H(t_o)$. [4]

The uncertainty of the universe's density leads to the uncertainty in the curvature of space. The curvature of space and the universe's fate depends on the value of density in the present day. This critical value of density of the universe is a function of time, defined as $\rho_c=\frac{3H_o^2}{8\pi G}$. For the convenience of further discussions, a density parameter $\Omega$ is introduced:

$$\Omega=\frac{\rho_o}{\rho_c}=\frac{8\pi G\rho_o}{3H^2} \tag{3}$$

Different values of dimensionless constant k would give different values of $\rho_o$ and $\rho_c$, respectively. [4] When k=1, the density $\rho_o$ exceeds the critical value, so $\Omega>1$. Then gravitational force will be sufficiently strong to halt the universe's expansion. In this model, the universe is expanding at a sufficiently slow rate that gravity will gradually slow the expansion and eventually stops it, causing the universe to contract and collapse; gravity bends space around itself, and space is finite in size but without boundaries. Such a model of the universe is referred to as the closed model. When k=0, $\rho_o$ equals the critical value of density and $\Omega=1$. In this type of model, known as the Flat model, the universe is expanding just fast enough to balance the gravitational attraction acting on it, so it would expand forever while the rate of expansion would gradually reduce; space is infinite and flat in this model. If k=-1, $\rho_o$ is less than the critical value $\rho_c$, which means that $\Omega<1$. In this universe model, the universe is expanding at such a high rate that the attractive force of gravity would never stop. The universe expands forever. Therefore, in this model, space is infinite and bent inward. This model is commonly referred to as the open model. [5]

The Flatness problem arises from the uncertainty of the value of $\rho_o$ due to observational constraints. More specifically, by only considering the luminous matter, the value of $\rho_o$ is approximately 10% to 20% of $\rho_c$. However, the existence of dark matter, a matter that does not interact with light, is supported by evidence such as the attractive force between two distant galaxies is higher than predicted, so the mass of each galaxy must be higher than conceived value. If considering the dark matter and luminous matter altogether, then the value of $\rho_o$ is about $\rho_c$, and $\Omega$ is about 1, which fits into the description of a flat model. [6] However, due to the difficulty in predicting the magnitude of dark matter and obtaining an accurate measurement of distances between galaxies, the uncertainty of $\Omega$ is high, lying between 0.01 and 2. Moreover, in order for $\Omega\approx 1$ at the present day, the value of $\Omega$ at some time at the early stages of the universe must be equivalent to 1 with high precision, and the density $\rho_o$ must be close to the critical value $\rho_c$ at the point of Big Bang. [7] It is puzzling why $\Omega$ has such a precise value and why the universe has such a close value of mass to the critical value near the point of the Big Bang. This puzzling question is known as the Flatness problem because when $\Omega=1$, the universe is flat. [3]

This paper aims to present the historical development of the Flatness problem, analyze past approaches to the Flatness problem, and consider arguments against the problem from the perspective of classical cosmology. In this paper, the Flatness problem is described quantitatively in section 2. Section 3 documents the history of the Flatness problem by referencing scientific literature. Section 4 examines past approaches toward the Flatness problem, and section 5 analyzes some arguments against the Flatness problem. The final section of the paper offers discussions of arguments and approaches regarding the Flatness problem and suggests possible fields for research.

## 2. A Quantitative description of the Flatness Problem

Rewriting equation (1) into another form, equation (4) is deduced.

$$\frac{\dot{R}^2}{R^2}+\frac{kc^2}{R^2}=\frac{8\pi G\rho_o}{3} \tag{4}$$

Applying the definition for $\Omega$ stated in equation (3), equation (1) can be written in terms of $\Omega$ .

$$\frac{\Omega-1}{\Omega}=\frac{3kc^2}{8\pi G\rho R^2} \tag{5}$$

Hence, an equation that relates the parameter $\Omega$ and the constant $k$ is obtained from the Friedmann equation (1).

$$\Omega=1+\frac{kc^2}{H^2R^2} \tag{6}$$

From this expression, it could be seen that if k=1, which is consistent with the Flat model, $\Omega = 0$ . On the other hand, if $k \neq 0$ (When k is equal to –1 or 1), as the direction of time is reversed back and approaches the point of the Big Bang (also known as the Big Bang Epoch), the value of $\dot{R}$ decreases, and the term $\frac{kc^2}{\dot{R}^2}$ would consequently decrease as well. Eventually, the value of $\Omega$ approaches one near the Plank Epoch, about $10^{-43}$ seconds after the Big Bang. Before the Plank Epoch, the universe's temperature was extremely high, so no current physical theories can describe the violent interaction between particles. [8] The fact that $\Omega$ is equivalent to 1 at the Plank Epoch means that the matter-energy density of the universe near the point of the Big Bang is nearly equivalent to the critical density, $\rho_c$. It is indicated that $\Omega$ must be precise, or at least to a high degree of accuracy, to equal one after the Plank Epoch. Therefore, because the current Big Bang theory cannot predict events before the Plank's epoch, the high degree of accuracy of $\Omega = 1$ is obtained immediately after the Plank time, a period when the universe is significantly dense. [9]

## 3. A summary of the historical development of the Flatness problem

In 1970, Dicke argued that if $\Omega$ is not equivalent to 1 to high precision in the early universe, the universe would quickly develop into a closed or open model; observational results do not suggest an open or closed universe (in fact, a nearly-flat universe is supported by observations). Therefore, $\Omega$ must equal 1 with high precision in the universe's early stages. [1] The Flatness problem initially received little attention compared to the horizon problem, the other drawback of the Big Bang model. [9] There were few published discussions on the Flatness problem other than the paper titled “The Anisotropy of Universe at large Times” published by Hawking in 1974; the paper addresses the precision of the mass density near the point of the big bang in an attempt to explain why the universe is isotropic. [10] However, after Guth proposed the inflationary universe model in 1981 to provide a solution to both the horizon problem and the Flatness problem, the Flatness problem was accepted as a legitimate scientific problem as it drew attention from the scientific community. [11] Several attempts were made to resolve this controversial problem, while the inflationary model, formulated by Guth, has been the most widely accepted solution and was adopted in several textbooks.

Two forms of Flatness problems have been presented, and many arguments are against both. The first form of Flatness problem, namely the qualitative flatness problem, is simply the initial argument presented by Dicke, which argues that a “high precision”, in other words “fine-tuning”, is required for $\Omega$ in the early universe for $\Omega$ to be nearly equal to 1 today. The second form of the Flatness problem is often referred to as the quantitative form, and it questions whether the observation results that $\Omega \approx 1$ at present requires a specific reason. The first form of the Flatness problem received many criticisms, such as from Coles and Ellis in 1997 and Lake in 2005. [12] They both write that under the Einstein de Sitter universe model, any deviations would increase over time, so it is natural that in the past, the parameter $\Omega$ was more precise or "fine-tuned"; thus they argue qualitative Flatness problem is invalid. [13]

Moreover, the extent of accuracy or "fine-tuning" needed is not defined clearly, which further reduces the legitimacy of the first form of the Flatness problem. On the other hand, some arguments against the second form of the Flatness problem have been proposed; Helbig, for instance, argued in 2012 that the quantitative Flatness problem does not exist in the context of classical cosmology does not exist in all Friedmann-Lemaˆıtre cosmological models. [14] Through analyzing models with different values of $k$ , he obtained the result that it is unlikely for the value of $\Omega$ at present to adapt to extreme values under any circumstances, meaning that it is unlikely for $\Omega$ to have differed significantly from 1; thus, Helbig argued that no explanation is needed for the observational result that $\Omega \approx 1$ at present, nor does the quantitative flatness problem exists.

## 4. Different approaches toward the Flatness problem

### *4.1. The Inflation Model*

Alan Guth proposed the inflation model in 1981 to resolve the Flatness problem and the Horizon problem. In the inflation model, the universe experienced a brief period of exponential expansion between $10^{-35}$seconds to $10^{-32}$ seconds after the Big Bang. After this period of rapid expansion (known as the inflationary epoch), the universe expands at a slower rate predicted by the standard Big Bang model. Guth proposed that the period of significant expansion exists because of a phase change. In its initial state, as proposed by the standard big bang theory, the universe was in a state of extremely high temperature and density, with all of its energy and masses concentrating on a significantly small point; as a result of the high energy state of the initial universe, forces were unified. [11] However, the universe cools as it expands, so the vital force was separated from the other forces and led to this phase change that caused the dramatic expansion of the universe in the inflationary period.

Assuming that the phase transition of forces takes place not immediately after the critical temperature for transition (above$10^{27}$ K) is reached, Guth discussed the supercooling state of the universe when the phase transition occurred. Supercooling refers to the state where temperature decreases to a value far below the phase-transition temperature. An astonishing state of the matter seems to come into existence as the universe supercooled far below the critical temperature; Guth refers to this state as the "false vacuum". Unlike ordinary matter, the density of the false vacuum would not decrease as volume increases because it is already in the state of the lowest possible energy density. This further indicates that, similar to the cosmological constant, the false vacuum's energy density will remain constant as the universe expand. An unprecedented result is obtained by combining Einstein's theory of general relativity with the characteristic of a false vacuum. When the universe is in the false vacuum state, the false vacuum causes a gravitational repulsion that causes the rate of expansion of the universe to accelerate. Based on this theory, the expansion scale factor R is considered to be approximately $10^{50}$ during the inflationary period. Recalling equation (6), because R is now equal to $10^{50}$, the second term on the right side of the equation is reduced to a factor of $10^{100}$, so the value of the term$\frac{kc^2}{H^2R^2}$ now approaches zero. Hence, it could be seen that $\Omega$ is equivalent to 1 with a significant precision under the inflationary model and that the Flatness problem is resolved. [11]

Although the inflation model is widely accepted as an effective solution to the Flatness problem, several problems are yet to be solved. Firstly, the inflation model suggests that today, $\Omega$ is still equivalent to 1 with a significant precision; however, observational evidence suggests a value for $\Omega$ at the present day that is about 0.2 or less. To understand the reason behind this discrepancy, the possibility of the influence on the value of $\Omega$ from undetected non-baryonic dark matter needs to be examined and the inflationary model, which is yet an undetailed theory, needs to be refined. [9]

### *4.2. Other discussions on the Flatness problem*

Other than the commonly embraced inflation model, various solutions have been proposed to resolve the Flatness problem. One of these attempts is the Time-Scale Argument. Tangherlini examined the flatness problem in a pulsating universe (a model deduced from the Einstein equations) in 1993; in the

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appendix, Tangherlini presents a time-scale argument demonstrating that the rate of expansion can be higher than the speed of light. [15,16] Helbig, in 2012, took the same approach of using timescale while considering the flatness problem in an FRW universe. By examining the relative time when under a universe that eventually collapses, Helbig concluded that there are no large values; thus, it was shown that the Flatness problem could essentially be avoided by using time-scale arguments. [14]

In 1994, Hu, Turner and Weinberg published the paper entitled "Dynamical solutions to the horizon and flatness problems"; they analyzed the two essential features of the inflationary model proposed by Guth, which are superluminal expansion and entropy production. [17] Through analysis, it was demonstrated that a satisfying dynamical solution to the flatness problem requires entropy production, a key feature of the inflationary model. The authors further discussed how the lack of this feature prevents those proposed adiabatic models from providing a complete solution to the problem. It was suggested by the end of the paper that because the inflationary model owns two features that are essential for the dynamical solution to both the horizon problem and the flatness problem, it may be the only dynamical solution to the flatness problem. Recently, Bamberger et al. published a paper 2018, exploring anisotropic scaling as a solution to the Flatness problem without relying on Guth's inflationary theory. [18] The group of cosmologists utilized the Horˇava-Lifshitz theory of gravity and the scaling property of this quantum gravity theory in order to solve for the solution of the Flatness problem. Eventually, it was concluded through deviations and calculations that a significant number of theories and models that satisfy the anisotropic scaling property of quantum gravity could all resolve the Flatness problem.

## 5. Arguments against the Flatness problem

While many cosmologists accept the Flatness problem as a legitimate problem, several argue that it is not a scientific problem; instead, it is a misunderstanding of the definition of a specific parameter. The Flatness problem has two essential arguments: the fine-tuning argument and the instability argument. As mentioned in the introduction, the fine-tuning argument suggests that there must be a reason behind the fact that $\Omega$ equals 1 with high precision in the universe during its initial stages. On the other hand, the instability argument suggests that if $\Omega$ does not equal 1 exactly, then it is impossible to observe that $\Omega \approx 1\ is$ in the present. Both of these arguments have been questioned by several studies. [19]

In 1995, Evrard and Coles argued that the Flatness problem does not exist in the framework of classical cosmology by adopting the approach of Bayesian interpretation of probability; they examined previous estimations of $\Omega_0$(The density parameter at the present day) that are based on inaccurate data, and it was inferred from these previous estimations that $\Omega_0$ lies in the range between 0.10 to 1.5. [20] Then, through analyzing the probability for the measure of density parameter $\Omega$ as $t$ approaches zero (the big bang epoch) based on previously known information. Eventually, Evrard and Coles drew the conclusion that the probability of the increasingly small intervals of $\Omega$ around unity does not necessarily to be small. Therefore no explanation is needed for $\Omega_0 = 1$; thus, they concluded that there is no flatness problem in the standard cosmological model. Most importantly, Evrard and Coles point out that $\Omega$ is not appropriate as a parameter. The same argument was also made by Lake in 2005, as he studied the changes of $\Omega$ in a dynamical universe that is constantly evolving. In the same paper, Lake further showed that the instability argument is not valid for the flat mode of the universe and the open model because they both expand forever; this is due to the reason of the significantly large value of $\Omega$ and the fact that the fine tune of the parameter $\alpha$ (a parameter relating the cosmological constant and the mass of a universe that is closed) needs to be fine-tuned to allow a non-flat universe and this parameter determines the type of model of the universe. [13] Because $\alpha$ is constant, the instability argument, which argues that $\Omega$ must be exactly equal to 1 in the early stages of the universe so that the universe is observed to be flat ($\Omega \approx 1$ ) in the present day, does not hold. Lake further asserted that the fine-tuning argument is not required to explain the observational result that obtains no large value of $\Omega$ due to the unlikeliness of such cosmological models with significant value for $\Omega$ .

Philip Helbig published a paper in 2012 in an attempt to examine the Flatness problem from the perspective of classical cosmology and evaluate whether the Flatness problem exists. Helbig considers the closed model ($k = 1$ and $\Omega > 1$ ) through calculations, and he also evaluates the two models that will expand forever (namely the flat model and the open model) by considering previous scientific papers on the same topic. In the end, Helbig concluded that the Flatness problem is, in fact, only a result of the way in which $\Omega$ is defined, and it does not exist for all Friedmann models. [14] Helbig made a similar argument to Lake, in which he also suggests that no fine-tuning is required to explain the fact that a large value of $\Omega$ cannot be observed because this high value only occurred in a certain short historical period in the universe.

## 6. Conclusion

The Flatness problem, initially formulated by Dicke and later populated by the suggestion of an inflationary model by Guth, is highly controversial nowadays since it is regarded as a legitimate problem by the majority, while many have argued that it is not a problem under the framework of classical cosmology. [21] Two main arguments have been presented in the Flatness problem. The first argument asserts that $\Omega$ must equal 1 with high precision in the early universe. Otherwise, the observed value of $\Omega \approx 1$ at the present day cannot be explained, and reasons must lie behind this high precision; this argument is often categorized as the qualitative form of the Flatness problem. This form of Flatness problem received large numbers of criticism, mainly because the extent of accuracy for $\Omega$ needed in the early universe has not been clarified and in an Einstein de Sitter model, deviations would increase over time. Hence, observing a more precise value for $\Omega$ in the early universe is natural. The second argument of the Flatness problem, often categorized as the quantitative form, argues that $\Omega$ must be exactly equal to 1 in the early universe. Otherwise, the observed value for $\Omega \approx 1$ at present cannot be obtained; this quantitative form essentially questions whether the $\Omega \approx 1$ at present should be explained. Although the quantitative Flatness problem has received far less criticism than the qualitative Flatness problem, Helbig argued in 2012 that the quantitative Flatness problem also does not exist because the value of $\Omega$ is unlikely to deviate far from 1 under different models with various value for k. Thus one should not be surprised that $\Omega$ is nearly equal to 1 at present. [15] At the same time, other approaches have been proposed as potential methods to resolve the Flatness problem, such as the anisotropic scaling utilized by Bamberger et al and the time-scale arguments first used by Tangherlini and later by Helbig.

The Flatness problem is highly significant simply because it is one of the major drawbacks of the standard big bang model (based on the Friedmann equations and Einstein's equations); the standard big bang model cannot explain the precise value of $\Omega$ at the early universe needed for the observed $\Omega \approx 1$ to be true. Thus, the answer to the reason behind the "fine-tuning" of the parameter $\Omega$ and whether this explanation is required is essential as it has the potential to improve current cosmological models. Therefore, more research is needed to thoroughly investigate the Flatness problem so that a clear answer to whether the problem is worth concerning and a developed solution can be proposed. The widely accepted inflationary model, only a framework instead of a detailed model, needs to be developed and refined. Moreover, cosmological models with different values of $k$ should also be examined in terms of the value of $\Omega$, so that a clear answer to whether an explanation for $\Omega \approx 1$ at the present day (in other words, whether the quantitative Flatness problem exists) can be provided. Despite many discussions on this topic, future progress on the Flatness problem is still needed. Thus, this essay calls for more future research on the Flatness problem in the scientific community to resolve the puzzle of the Flatness problem.

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